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Modelling of composite materials energy by fiber bundle model★

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Abstract. In this paper, the fiber energy in composite materials, subject to an external constant load, is studied. The investigation is done in the framework of fiber bundle model with randomly oriented fibers. The charge transfer is done only between neighboring close fibers according to the local load sharing. During the breaking process, the fibers expand, increasing their elastic energy, but when the fiber breaks, it loses its link with its neighboring fibers reducing the cohesive energy of the materials. The results show that the material energy presents one maximal peak at cross over time which decreases linearly with the applied force and scales with the lifetime of the material. However, the temperature does not have a remarkable effect on the material energy variation. In addition, the link density fiber decreases exponentially with time. The characteristic time of the obtained profile decreases with the applied force. Moreover, this density decreases with applied forces according to the Lorentz law with a remarkable change at critical force value.

1 Introduction

The failure process in materials and structures is a substantial phenomenon involved of in industry, technology and scientific community. The failure process in materials cannot always be represented by a basic linear equation due to the disorder and intrinsic non-uniformity in materials. Therefore, the theoretical models of statistical physics are employed generally to study the characteristics of the failure phenomenon and their microscopic mechanism [1]. Accumulation of micro-cracks can produce the macroscopic material failure, which corresponds to the thermodynamic limit of discrete approaches. In spite of the benefits of composite materials, they are obviously more complicated to simulate owing to their different geometrical armatures. Composite materials characteristically have three scales of importance: microscopic, mesoscopic and the macroscopic. The last one constitutes the totality of the system, so the composite is an assembly of fibers contained in a matrix [2].

As the aforesaid literature investigation denotes, the fracture process of bearing failure in a sealed material junction is much intricate. One origin for such intricacy may be attributed to the besides applied load in the surrounding of the omission. In addition, fiber micro-fracture, matrix breakdowns, delamination, and other types of failure regularly serve as the 3D fracture way below the applied force in materials. On that account, a comprehension of the emergence and advancement of the microscopic failure is crucial to the assessment of the parameters of failure resistance in the fiber bundle [3].

Moreover, the fluctuation relative to the average quality represents an important part in the characterization of the failure phenomena. On the macroscopic scale, several elongation and failure kinds of recent composites are presented by the force–strain correlation. More than that, the rupture characteristics of composites are intuitively categorized as brittle, semi-brittle, and plastic. In the elastic failure mechanism, the material submits a transition from a stage of local fracture to a stage of global fracture below critical applied load [4]. Overall, the important force, major individual fibers that are employed in composite materials are delicate, possessing elastic elongations that shall be described statistically. Every approach for the elastic elongation of materials regrouping some fibers shall consider the dispersion in their fracture force stages so as to have most pertinence [5].

The dispersion in fiber resistance leads the crack of composite material, exposed to elastic force, appears once the fibers have been fractured into extents. So we should mention that any augmentation in imposed force cannot be transferred to the fibers owing to the fact that the boundary of the matrix or the interface fleece has been achieved [5].

Almost statistical researches on the failure in composite materials depend on a pretended fiber bundle model (FBM), which in almost all studies, may properly catch the different static and dynamic characteristics of the rupture

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process in our systems [6,7]. The mechanism of the FBM is so elementary that it is plenty simple to acquire certain conclusions analytically or exact statistical characteristics numerically. Fiber-bundle model is an adequate tool for modeling the fracture of composite materials from a micro-scale. This model is composed of a number of components called fibers disposed in random directions and under an applied load value, either constant or time-dependent. The fracture characteristics of the components are statistically delivered depending on a precise probability distribution, and the purpose is to evaluate the failure characteristics of the entire system [6–11].

Classical FBM is the study of a bundle of parallel elastic fibers which irreversibly breaks when the force surpasses a definite threshold value. To model the failure phenomena of different materials below force verified control conditions, the imposed charge is quasi-statically enhanced; in particular, merely the fragile intact fibers fail after each stage of the charge enhancement. The released charge from fractured fibers will be shared among the surviving fibers depending on a sharing rule, thus generating a further rupture.

It was understandable from the beginning that FBM may be built to model either the static resistance of composites or their rupture by strain. The first kind of the FBM endeavor to attempts to imitate laboratory essays where specimens (our bundles) are exposed to an enhancing force and the goal is to assess the modicum force requisite to fail the entire system. The second kind of FBM modeling fracture by overdue strain: each fiber is attributed an initial time to fracture which relies on the immediate force on each fiber and therefore on anterior force. The first kind of model is famous as static FBM and the second kind as time-dependent or dynamic FBM [6].

One of the important elements of FBM, both static and dynamic, is the load sharing rule. As any fibers fail, the force that they were carrying is shared to other surviving fibers in the system. According to which fibers take this force, three elementary load sharing mechanisms are designed: local load sharing (LLS), global load-sharing (GLS) and hierarchical load sharing.

The first mechanism, where the supplemental applied loads are distributed in the proximate board of the intact fibers, is recognized as LLS rule, and this has demonstrated widely more complications to nominate from an analytic viewpoint. In this kind of load transfer merely the neighboring intact edge reaches the maximum force of a broken fiber. This load sharing scheme is beneficial for simulation of composite materials where friction between components minimizes the re-sharing of load to a local section, and has found many applications in the manufacturing of composite materials [6].

In our approach an assembly of identical fibers is considered, which is then exposed to a constant applied force. The variation of the system is operated by thermal noise: the applied load imposed on our system has thermally provoked fluctuations which may induce failure. The main benefit of this elementary approach is that various important macroscopic features of the system may be solved analytically. As a result, it was proved that the lifetime of the system has Arrhenius-type reliance on the force and especially on thermal noise when the fibers have a random elongation [12].

Although, the microscopic scale simulation is an essential implement to get results concerning the stochastic dynamics of thermally provoked failure. A Computer simulation where thermal noise variation provokes the stress fluctuations presents various restrictions on the disposable interval of temperature, on force values and on the system size [12]. Mechanically linked junctions have mainly been used for the important charged material constituents. However, the elasticity of composites may provoke complications of gap elongation below compressive force.

2 The used model: Fiber bundle model

The considered materials are more realistic than the ones used in the classical FBM model with parallel arranged fibers. Each fiber i is associated with a random constant direction angle θ_i according to the vertical axis. Hence, the matrix ensures the cohesion of the system and each fiber keeps the same direction during the time load application (constant θ_i). In the used model, a fiber bundle of size L consists of a large number $N_0 = L \times L$ of fibers randomly inclined relative to the vertical axis (OZ) with a constant angle θ_i taking values of range $(0 \prec |\theta_i| \prec \frac{\pi}{2})$ according to uniform distribution:

$$P(\theta) = \int_0^\theta p(y)dy = \theta. \tag{1}$$

Each single fiber is subject to an imposed tensile constant force f which ensures an axial displacement x_i linked by the Hooke's law [1]:

$$f_{\theta_i} = k \cdot x_i \tag{2}$$

where k denotes the stiffness, which is the same for all the fibers.

However, the sinus component is mitigated by the matrix-fiber interface reaction. For this reason, the mean action is arising from the cosine force component.

On the other hand, and in order to take into account the temperature effect, the corresponding local elongation ζ of each fiber is introduced [13]:

$$\xi = \gamma l_i \sqrt{K_B T} \tag{3}$$

where K_B is the Boltzmann constant; γ is coefficient of proportionality between elongation and square root of one-dimensional thermal energy (fiber); l_i is the initial length of the fiber.

The increase of temperature will affect the lifetime of the system [13–15]. Then, the actual total arising length x of each fiber is written as:

$$x = x_i + \zeta. \tag{4}$$

If the arising length of fiber i exceeds the breaking threshold elongation x_c, the fiber cracks and its load is shared by its neighboring remaining fibers [1,16]. In this

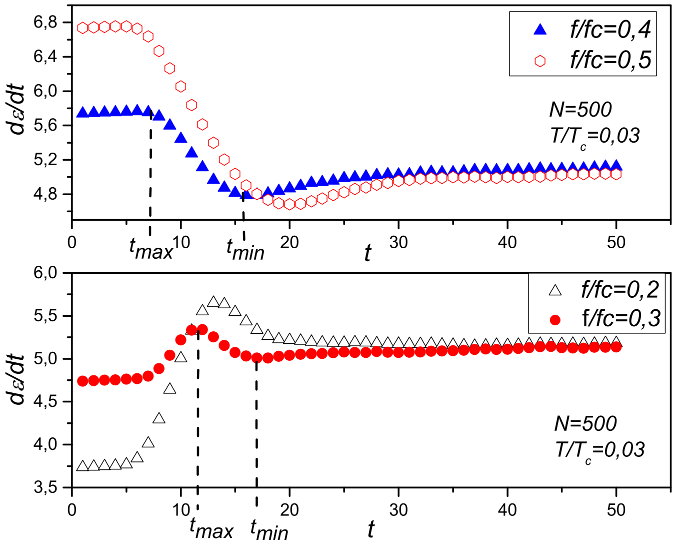

Fig. 1. Time evolution of the mean elongation of fibers per time unit.

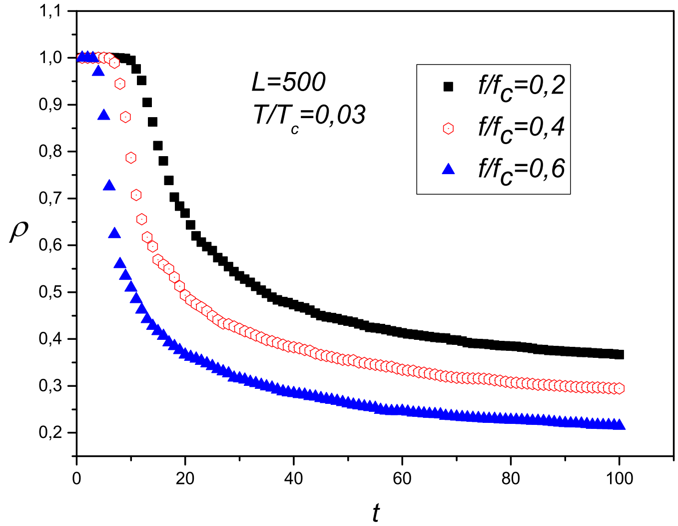

Fig. 2. The time evolution of the link density of fibers for different applied load values.

paper, the breaking threshold elongation x_c is randomly chosen and given a density of probability $p(x)$ associated to the uniform distribution $P(x)$, as presented previously in equation (1).

During the failure process, when some threshold values of stretching exceed the stress carried by the broken fibers, they are shared by the intact neighboring fibers and so on. of course the fibers which are newly exposed to the load, say, after an avalanche, have a relatively low load compared to the ones which are accumulating the load shared from the earlier failures and are still surviving [1]. Thus each of the other fibers undergoes another elongation δl due to the force released by the LLS share and the expression of total elongation x' of each fiber becomes:

$$x' = x + \delta l. \quad (5)$$

3 Numerical results and discussions

In our investigation, N fibers oriented randomly make a bundle with a square lattice of size LxL. Each fiber of this bundle is subject to an external load f and when its elongation value exceeds its threshold elongation, the fiber cracks. However, the failure process in composite materials can be characterized by calculating the time evolution of the mean elongation of fibers per time unit $\frac{d\varepsilon}{dt}$. Corresponding results for different values of applied load are plotted in Figure 1. This parameter exhibits three different stages for both low applied load and high applied load. In the first stage the mean elongation fibers per time unit increases over time until reaching a maximal value at crossover time $t_{\max}$. This increasing process is clearly observed in the high applied load. After this maximal value, $\frac{d\varepsilon}{dt}$ decreases in the second stage where the fibers are critically self-organized by reaching a minimal value at the crossover time $t_{\min}$. Finally, in the third stage, the fibers are still elongated until their final crack process.

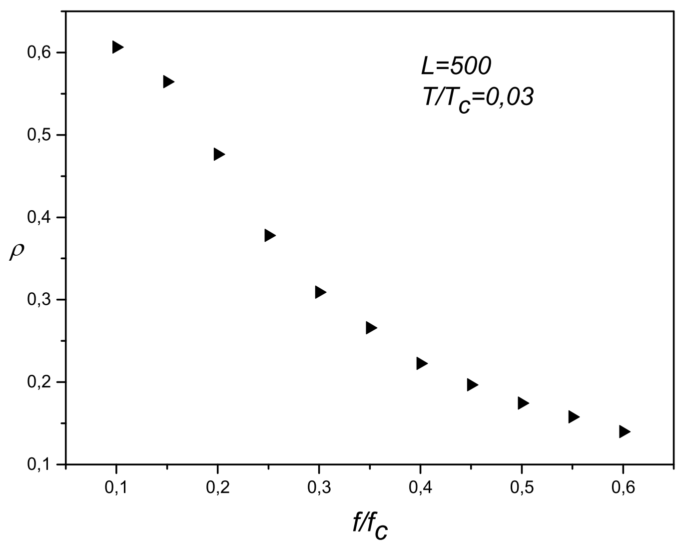

Fig. 3. The link density of fibers versus applied load.

During the breaking process of composite materials, some fibers become disconnected with their neighboring ones, and the system is presented as a cluster of fibers with different sizes. Therefore, we can characterize this breaking process by calculating the link density $\rho = \frac{Nl}{N_0}$ which assures the connection between neighboring intact fibers. N_l and N_0 are the link number between intact fibers at time t and at initial time respectively. The time evolution of the link density ρ for different applied load values is plotted in Figure 2. The threshold time, when the crack process is started, decreases with increasing applied load. Moreover, the link density decreases exponentially with time. This result is more consistent with the one investigated by the percolation theory [17]. Hence the obtained profile of the link density ρ is similar to the one of the broken density [18]. The link density profile versus applied load for a temperature value $T = 0.03\,T_c$ is plotted in Figure 3. The obtained profile is best fitted by a Lorentzian law with

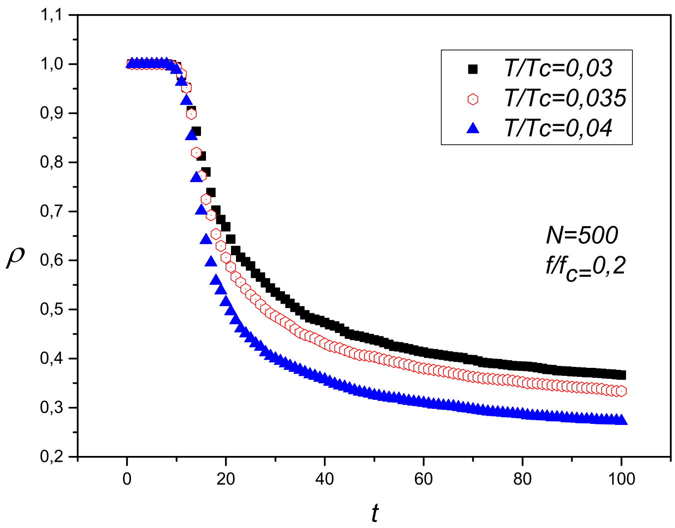

Fig. 4. The time evolution of the link density of fibers for different temperature values.

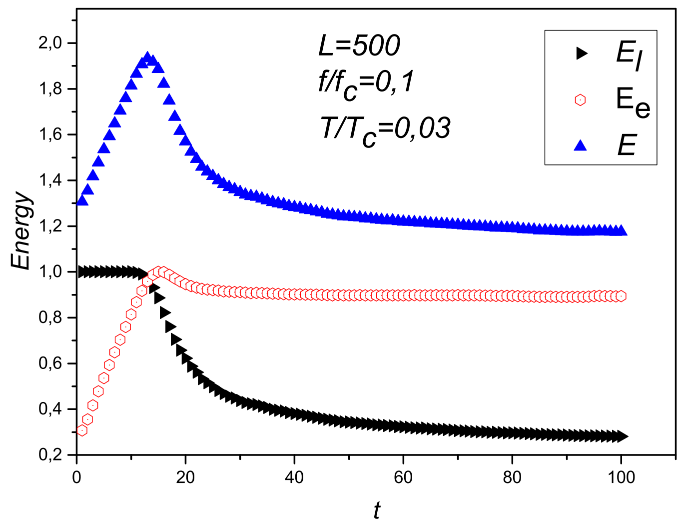

Fig. 5. The time evolution of the material energy.

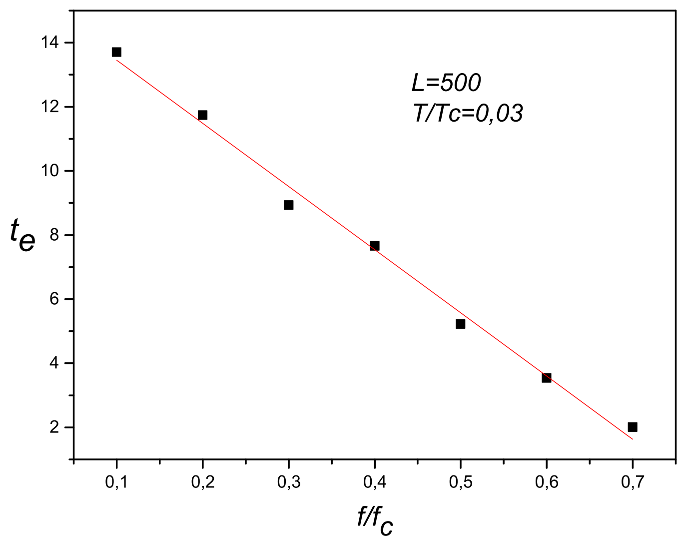

Fig. 6. The crossover time t_e versus applied load values.

inflexion point at $f = 0.3 f_c$. This result is consistent with the one in reference [19] where the authors have investigated the mean elongation fibers versus applied load.

Thermal performance is one of the principal properties of composite materials which controls and determines the optimal conditions where the materials can resist thermal noise. Hence, to show the importance of this parameter, the time evolution of the link density for the applied load value $f = 0.2 f_c$ is calculated. The corresponding results for different temperature values are plotted in Figure 4. The temperature has no remarkable effect on the threshold time when the crack process starts. But, it presents a similar behavior to the applied load.

Piezo-composites materials based on a piezoelectric polymer and ceramic, are promising materials due to their excellent properties with piezo-passive-dampers, including piezoelectric ceramic particle, polymer, and a carbon black, which suppress the noise vibration more effectively than traditional rubbers.

However, by neglecting the energy due to the matrix-fibers interaction, the total material energy is defined by the sum of two parts:

$$E = E_e + E_l. \tag{6}$$

For an elastic material under a force, the elastic energy may be related to the elongation. This energy may be obtained from the half of multiplication of the fibers stiffness by the square of fibers elongation produced by the charge [19]. The elastic energy express as:

$$E_e = \tfrac{1}{2} k \cdot \delta l^2 \tag{7}$$

where δl is the fiber mean elongation.

Moreover, the second part of total energy is the energy links assured by the link fibers:

$$E_l = N_l \cdot E_0 \tag{8}$$

where E_0 is the single link energy.

In Figure 5, the time evolution of the three aforementioned energies is presented for an applied load value $f = 0.1 f_c$ and a temperature value $T = 0.03 T_c$. The elastic energy E_e and the link energy E_l present a two regimes variation, separated by a crossover time t_e. In the first regime where $t < t_e$, the elastic energy increases linearly with time due to the elongation of fibers subject to external load. But, the link energy is constant timely because all fibers are still surviving. However, in the second regime where $t > t_e$, some fibers crack, hence these two energies decrease with time. Additionally, the total material energy E presents a maximal value at the crossover time t_e which decreases linearly with applied load (see Fig. 6). Moreover, the crossover time t_e can describes the crack process in composite materials because it scales with the life time of the material t_f as: $t_e = 0.2 t_f$.

In the first regime $t < t_e$, the applied load leads to an increasing process of the material energy, but when the

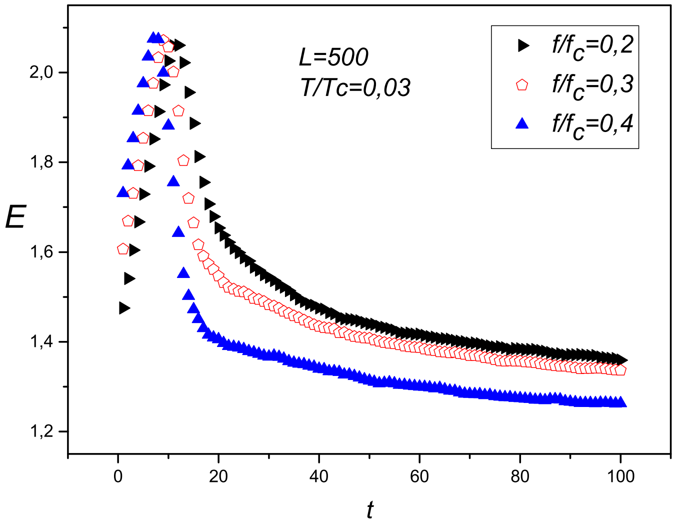

Fig. 7. The time evolution of the total material energy for different applied load values.

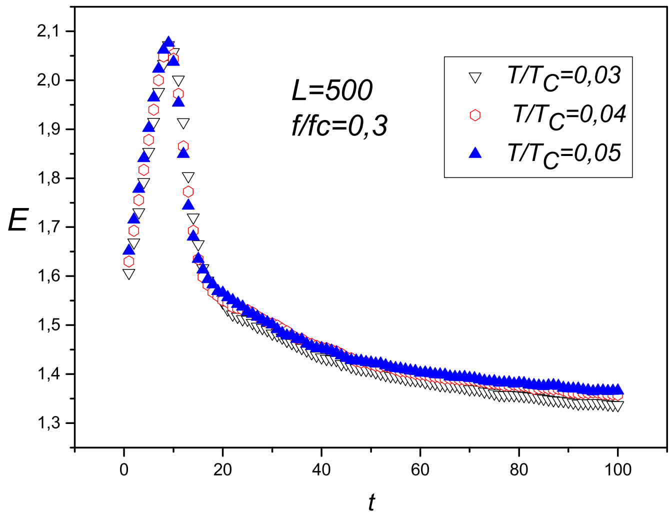

Fig. 8. The time evolution of the total material energy for different temperature values.

fibers start cracking, the material energy decreases with applied load (see Fig. 7).

To check the temperature effect on the material energy, the total material energy E for different temperature values is calculated. The corresponding results for an applied load $f = 0.3f_c$ are plotted in figure 8. Thermal process has no remarkable effect on the total material energy. This result is consistent with the one investigated in [20–22].

4 Conclusion

In summary, the energy aspect in composite materials was investigated using the Fiber Bundle Model with local load sharing rule. The fibers are considered to be randomly oriented. The results show that the materials energy is presented with two different parts. The first one is corresponding to the elasticity process of the material when it's subject to an external load, and the second is due to the link process between fiber clusters. However, these two energies present two different regimes with opposite behaviors. Additionally, the total material energy exhibits a maximal value at crossover time which decreases linearly with applied load but it is less dependent on the thermal process. The results show that this crossover time scales with the material lifetime. Moreover, the fiber elongation density has been studied. The results show that this parameter exhibits three different stages for both low and high applied loads. The intermediate stage corresponds to a state where the fibers are critically self-organized. Moreover, our investigation shows that the fiber link density decreases exponentially with time where the characteristic time of the obtained profile decreases with applied load.

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Author contribution statement

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