Eur. Phys. J. Appl. Phys. (2016) 73: 10402
DOI: 10.1051/epjap/2015150367

THE EUROPEAN PHYSICAL JOURNAL
APPLIED PHYSICS

Regular Article

Evaluation of electrical transverse conductivity of the unidirectional CFRP*

Mohamed Khebbab[1,2,3,a], Mouloud Feliachi[2], and M. El Hadi Latreche[3]

[1] Université de Bordj Bou Arréridj, 34265 Bordj Bou Arréridj, Algeria
[2] IREENA-IUT, CRTT, 37 boulevard de l'Université, BP 406, 44602 Saint-Nazaire Cedex, France
[3] Laboratoire LEC, Université Constantine 1, route Ain Elbey, 25000 Constantine, Algeria

Received: 18 July 2015 / Received in final form: 14 August 2015 / Accepted: 23 November 2015
Published online: 13 January 2016 –

Abstract. In this paper, a technique for the calculation of the electrical transverse conductivity of unidirectional carbon fiber reinforced polymer (CFRP), based on Markov chains, is proposed. Inspired by the microscopic cross-sectional structure of CFRP, an electrical percolation system is constructed. The effective transverse conductivity is derived from an equivalent conductance of the percolation network. To achieve such a determination, a notion of escape probability associated to absorbing Markov chains is applied. The obtained results are compared with those given by percolation theory; and also with published experimental data. Our results are shown to be in good agreement with the references.

1 Introduction

The search for new materials combining stiffness, strength and lightness to be used in different types of structure is an active field of research. Materials such as carbon fiber reinforced polymer (CFRP) seem to be a good choice. The CFRP are made by the impregnation of carbon fibers, that are electrically conducting, in an insulating resin matrix. The knowledge of the resulting electrical conductivity is of great importance in most applications. An experimental approach can be used to characterize this new conductivity. One idea is to inject an electric current flowing through the composite material [1]; measurement of potential drop allows to derive the conductivity. However this method suffers from the need to use many measurement samples for getting sufficient accuracy. In mathematical modeling, researchers have proposed several numerical methods to study CFRP materials. Some of them represent CFRP by a resistance network, including two kinds of resistance R_L and R_c. R_L is the resistance in the longitudinal direction, calculated for a length called "electrical effective length", noted δ_{ec} defined as the average distance between adjacent contact points. R_c is the inter fiber contact resistance [2,3].

In reference [4], an analytic expression is used to evaluate a set bounds of the transverse resistivity of two predefined network configurations. In reference [3] the position of inter fiber contacts was studied using random contacts of line segments set of length δec. Other works use homogeneization methods to get the effective conductivity [5]. Those methods involve replacing a heterogeneous medium by a homogeneous one, having the same macroscopic behavior; those methods still use measurement information at least for the first layer of the CFRP sample.

In our work we are interested in characterizing the transverse electrical conductivity of a unidirectional CFRP. In spite of the fact that the fibers are aligned, there remain random contacts between them; this is essentially due to fiber waviness and lack or absence of insulating matrix resin between them. This will cause non-vanishing conductivity transverse to the fibers. Our approach is based on the idea of modeling CFRP by a resistance network, starting from the observation of the number of contacts on one fiber, seen in a microscopic transverse section. To represent a random distribution, a fiber removal model was adopted.

2 Network representation

The knowledge of the number of fiber contacts and their locations is of great importance, since it will determine the path of the electrical current through the CFRP. There are two ways to represent random fiber distributions in a unidirectional CFRP.

2.1 Monte Carlo fiber placement

The fibers are considered as circles placed in a rectangular box called "Cell". Each circle is described by two random

[a] e-mail: mohamed.khebbab@univ-nantes.fr

* Contribution to the topical issue "Numelec 2015 – Elected submissions", edited by Adel Razek

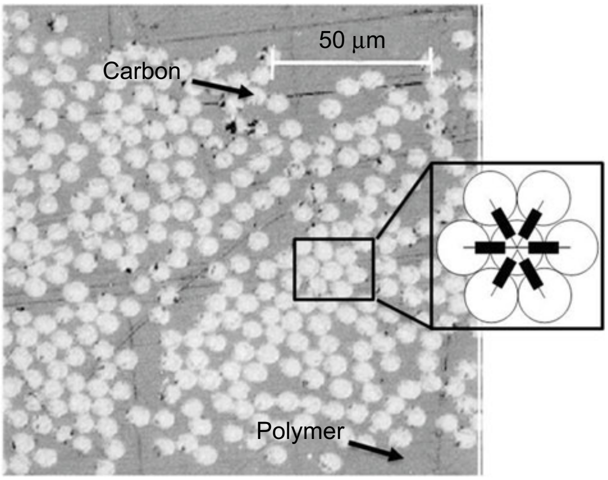

Fig. 1. Transversal section of unidirectional fibers.

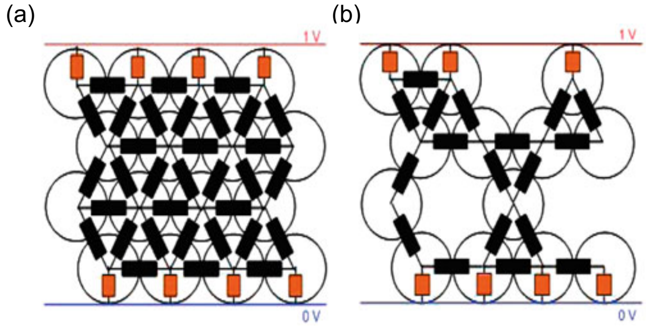

Fig. 2. Resulting network: (a) fully filled, (b) partially filled.

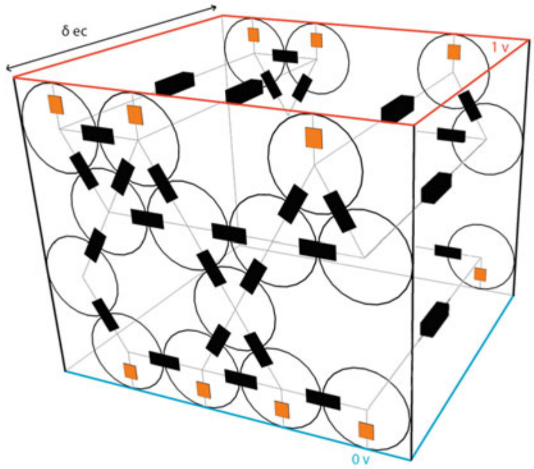

Fig. 3. Example of a resistance network cell.

numbers which represent center coordinates. The circle generation continues until the desired volume filling ratio is obtained. Taking care that every new circle does not overlap the previously generated ones (10% overlapping is generally allowed) and does not cross the cell limits [6].

2.2 Monte Carlo fiber removal

In this model, we first fill the cell in an orderly way; then we randomly remove circles until we get a desired volume filling ratio. Besides its simplicity and speed, this technique has the advantage to warrant filling rates higher than 60%. Based on this, a cross-section of unidirectional composite is taken as a starting point to form our network resistances. The CFRP layers are considered to be formed by a perfectly a straight cylinders of carbon, oriented in the same direction.

In reference [2], it is reported that the inclination angle of the unidirectional fiber composite is between 0.2° and 1° with a ripple around its axis on distance δ_{ec}. Figure 1 shows a picture of unidirectional CFRP material taken by scanning electron microscopy (in the GEM laboratory, Saint-Nazaire) with possible contacts between fibers. In this context, we construct our resistance network as shown in Figure 2. By numbering nodes (circle centers) and connecting them we will get a connected graph.

3 Calculating effective conductivity

3.1 Longitudinal conductivity

The longitudinal conductivity (parallel to the fibers) is due to current flowing along the fibers, a good approximation of its value is given by a simple mixture law [7]:

$$\sigma_{\text{long}} = \tau \times \sigma_{\text{carb}}, \tag{1}$$

where τ is the filling rate, σ_{carb} is the carbon conductivity (S/m).

3.2 Transverse conductivity

The conductivity perpendicular to the fibers is due to inter-fiber contacts that depend on the volume fraction of the fibers and also on the degree of inter-fiber contact [8]. In this section we introduce our representative volume element, in which the transversal conductivity is calculated. Due to the random distribution, the fiber contacts are not necessarily located in the same plane; however this fibers arrangement was used with proper δ_{ec} adjustment in analytical expression to fit experimental results [2].

Based on this we can consider contacts as shown Figure 3, where contacts are in the same plane.

Once we have defined our representative volume element, the analogy with electrical networks as undirected connected graphs is considered [9]; with edges xy_i of conductance C_{xy_i} between the node x and its neighboring nodes y_i.

The voltage V_x at node x can be calculated:

$$V_x = \sum_{i \in N} \frac{C_{xy_i}}{C_x} V_{y_i}, \tag{2}$$

where C_x is the sum of the conductances connected to node x and N the number of nodes connected to the node x (Fig. 4).

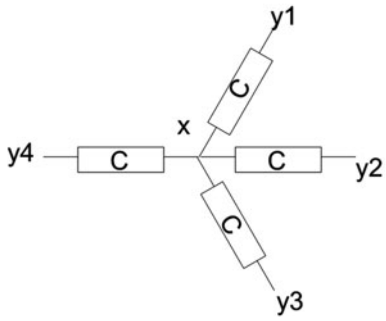

Fig. 4. Example of connection between node x and nodes yi.

We define a matrix P as a transition matrix associated with our circuit Figure 3; whose (i, j)th entry is p_{ij}, that is the probability of a transition from i to j is defined as:

$$P_{x \to y_i} = \frac{C_{xy_i}}{C_x}. \quad (3)$$

The calculation of potentials will be done by Markov absorbing chains. A Markov chain is absorbing if it has at least one absorbing state and from every state it is possible to go to an absorbing state, by definition:

- Absorbing state: once the walker reaches this state, it is impossible to leave it. The probability of transition is $p_{ii} = 1$ (nodes connected to generator).
- Non-absorbing or transient state: applied to the rest of nodes.

We reorder the states so that the absorbing states come first and the non-absorbing states come last:

$$P = \begin{bmatrix} I & O \\ R & Q \end{bmatrix}, \quad (4)$$

where we introduced:

- I: an identity matrix indicating that one always remains in an absorbing state once it is reached.
- O: a zero matrix representing 0 probability of transitioning from the absorbing states to the non-absorbing states.
- R: the transition probabilities from the non-absorbing states to the absorbing states.
- Q: the transition probabilities between the non absorbing states.

Knowing those matrix we can calculate the matrix of absorption probabilities, B, as:

$$B = (I - Q)^{-1} R. \quad (5)$$

The coefficients, b_{ij}, of this matrix B give the probability of ending up at an absorbing state j starting from the transient state i, which is the probabilistic definition of voltage.

The number of rows i in matrix B corresponds to the number of transient nodes, and the number of columns j corresponds to the number of absorbing nodes.

In order to compute the transversal conductance, we introduce two additional nodes:

Node a, connected with infinite conductivity to all nodes representing fibers touching the $1V$ plane, and node b, connected with infinite conductivity to all the nodes representing fibers touching the $0V$ plane. The potential distribution over the nodes can be computed from random walks from node a to node b. With the computed potential distribution, the equivalent conductance follows as:

$$C_{\text{eff}} = \frac{i_a}{v_a}, \quad (6)$$

where i_a is the current flowing into the network from node a. For $v_a = 1$ this current is given by:

$$i_a = \sum_{i \in N} (V_a - V_{y_i}) C_{ay_i}. \quad (7)$$

Multiplying by the sum of total conductance connected to the node a i.e., C_a:

$$i_a = \sum_{i \in N} (V_a - V_{y_i}) C_a \frac{C_{ay_i}}{C_a}. \quad (8)$$

Define P_{ay} as the probability assigned to the walk on selecting edge ay_i at node a, knowing that $C_a V_a$ does not affect the summation equation (7):

$$i_a = C_a V_a - \sum_{i \in N} V_{y_i} P_{ay_i} C_a. \quad (9)$$

For $V_a = 1$:

$$i_a = C_a (1 - \sum_{i \in N} V_{y_i} P_{ay_i}), \quad (10)$$

from which we define the Escape probability as:

$$P_{\text{esc}} = 1 - \sum_{i \in N} V_{y_i} P_{ay_i}. \quad (11)$$

We get:

$$i_a = C_a P_{\text{esc}}, \quad (12)$$

we deduce the equivalent conductance:

$$C_{\text{eff}} = C_a P_{\text{esc}}. \quad (13)$$

Having the equivalent conductance and the dimensions of the cell, the effective transverse conductivity will be:

$$\sigma_T = \frac{C_{\text{eff}} l}{S}. \quad (14)$$

4 Simulation results

The simulation is performed on an initial network of 100 fibers with a diameter of 6 μm, the inter fiber contact resistance R_C is taken as the resistance of a section of fiber length equal to the diameter [4]. For each filling rate several circuit configurations are possible, and for each configuration the effective conductivity is evaluated. Then, the average transverse conductivity is determined.

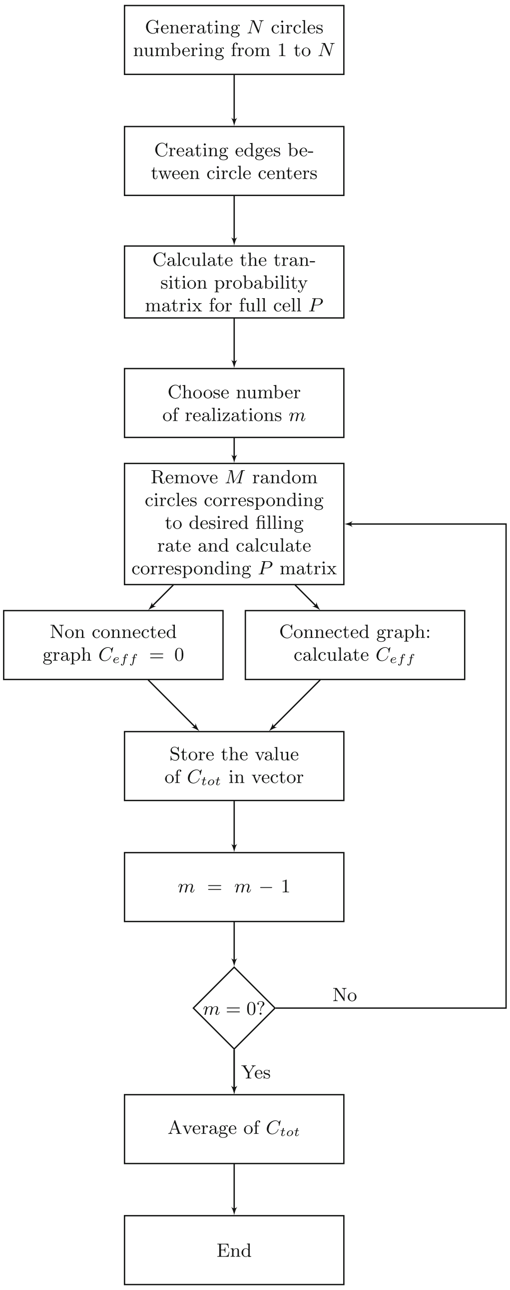

Fig. 5. Flow chart describing the implemented algorithm for one desired fiber volume fraction.

It is to be noted that for lower filling rate, below 35%, there will be many non-connected graphs, so the electrical conductivity will be zeroed, but for rates above 35% the percolation threshold is reached; so we have less chance to get a non connected graph. The algorithm is implemented in Matlab as described in Figure 5.

Taking the parameters, fibers electric conductivity 66 kS/m and $\delta_{ec} = 320$ μm. A comparison with results determined from percolation theory [10] is carried out (Fig. 6) where:

$$\sigma = \sigma_c + (\sigma_c + \sigma_M)\left(\frac{\phi - \phi_c}{F - \phi_c}\right)^t, \qquad (15)$$

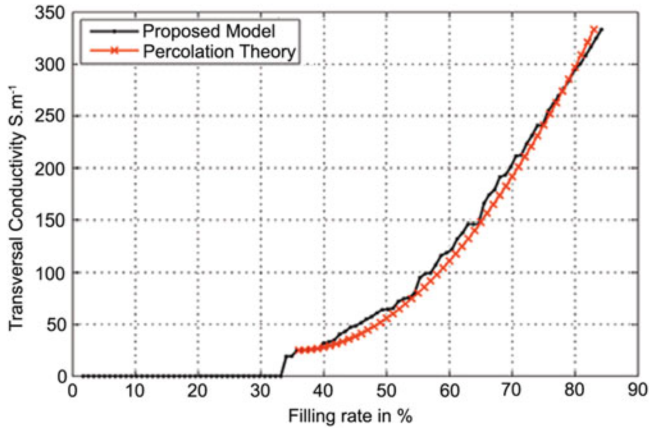

Fig. 6. Effective electrical conductivity variation with respect to the filling rate.

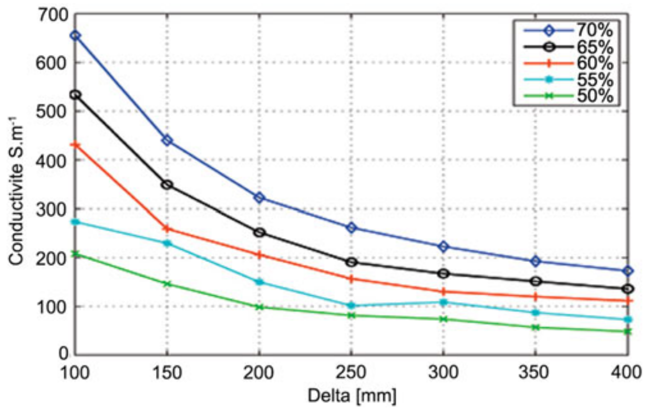

Fig. 7. Effect of the average distance between two contacts on the transverse conductivity.

ϕ: volume fraction,
ϕ_c: critical fiber volume fraction,
F: the maximum volume fraction,
σ_c: critical conductivity,
σ_M: maximal value of the electric conductivity,
σ: effective conductivity,
t: setting range 1.6–1.9.

The ϕ_c (percolation threshold for CFRP) is generally found in the literature are between 35% to 45%. and σ_M is admitted between 65 and 70%, so we power law relation was used to approximate their value $\sigma_M = (\phi - \phi_c)^t$ which include only exponential part of conductivity. To see if our model feet with percolation theory, an additional couple (ϕ_c, σ_M) is add, this couple correspond to the theoretical maximum volume faction reached by monte-carlo-removal model which is around 80%.

The percolation theory model requires experimental determination of the percolation threshold and the maximum critical conductivity, while the approach we propose is based on simple considerations of a resistance network and current paths. Our model predicts 140–160 S/m for a filling rate of around 60% for unidirectional CFRP which corresponds well with the experimentally obtained values given in reference [11]. Another important parameter that influences the transverse conductivity is the average distance δ_{ec}; which we have used for calculating the longitudinal conductance in our representative volume element.

Figure 7 shows the inverse effect between average distance δ_{ec} for different volume fraction, and the transverse effective electrical conductivity. We can see that when the distance between two successive contacts increases, the effective transverse conductivity decreases in low value even for high volume filling rate.

5 Conclusion

In this paper, the electrical conductivity in the transverse direction of unidirectional CFRP material is numerically calculated. Based on the distribution of fibers in the insulating matrix viewed in transverse microscopic scale, an original percolation conductance network is obtained. The equivalent conductance is evaluated. Absorbing Markov chains are used for calculating voltages at different nodes of the obtained connected graph, and the escape probability is used to get equivalent conductance of the obtained resistance network. the influence of the main parameters on this transverse conductivity which are filling rate and electrical effective length are also shown. A good agreement with results found in the literature is observed.

References

1. R. Lange, G. Mook, NDT&E International **27**, 241 (1994)
2. J.B. Park, T. Okabe, N. Takeda, Smart Mater. Struc. **12**, 105 (2003)
3. G. Wasselynck, D. Trichet, J. Fouladgar, IEEE Trans. Magn. **49**, 1825 (2013)
4. A. Gillet, P. Olivier, A. Al Maghribi, I. El Sawi, *Prediction of Electrical Properties of an Aeronautic Composite* (JNC16, Toulouse, 2009)
5. S. Bensaid, D. Trichet, J. Fouladgar, IEEE Trans. Magn. **45**, 1859 (2009)
6. K. Shimada, Physically-based mesh generation: automated triangulation of surfaces and volumes via bubble packing, Ph.D. thesis, Massachusetts Institute of Technology, 1993
7. C. Thiagarajan, I.M. Sturland, D.L. Tunnicliffe, P.E. Irving, Proc. SPIE **2361**, 128 (1994)
8. J.B. Park, T.K. Hwang, H.G. Kim, Y.D. Doh, Smart Mater. Struc. **16**, 57 (2007)
9. S.J. Laurie, P.G. Doyle, *Random Walks and Electric Networks* (The Mathematical Association of America, Washington, DC, 2006)
10. D. Stauffer, A. Aharony, *Introduction to Percolation Theory* (Taylor and Francis, London, 1992)
11. A. Piche, I. Revel, G. Peres, F. Pons, Numerical modeling support for the understanding of current distribution in carbon fibers composites, in *Electromagnetic Compatibility – IEEE, Athens, 2009*, pp. 1–4