

# A simple method for determining the thermal effusivity of defects

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**Abstract.** In this article, a simple method was proposed for determining the thermal effusivity of defects using pulsed thermography. This method is complementary to other methods such as those that implement periodic thermal stresses for identification of the thermal properties of materials, or which use specific photothermal analysis or extended. Theoretically, this technique is based on solving the heat equation in the case of an isotropic solid heated by a delta Dirac pulse. The application conditions of this new technique have been studied by numerical simulations. A finite element modeling has been performed in the case of a steel sample which may contain geometry defects filled in by air, resin, oil, wax or water. The analysis of surface temperature distribution of the infected samples by the proposed method allowed to determine the thermal effusivity of defects. The experimental measurements have confirmed the importance and simplicity of this method in the characterization of defects using pulsed thermography. The results show the possibility of approaching, at an acceptable error, the defect thermal effusivity.

## 1 Introduction

Pulsed thermography is a nondestructive evaluation method, which has been qualitatively and quantitatively applied to different types of materials to detect a variety of defects, such as corrosion and delamination in composites and metals, etc. [1]. The application of pulsed thermography is used to extract effectively the size and depth of the defect [2–6].

To apply the pulsed thermography, the tested material needs, on its surface, a stimulation by a heat pulse and analyze its self-cooling process [7, 8]. During this process, the rate of temperature decrease from a point above the defect is different from a point above the defect-free zone. This variation is caused by a difference in values of thermal effusivity of the material and its internal defect.

The thermal effusivity of a material characterizes its ability to exchange thermal energy with its environment. It is given and defined as:

$$e = \sqrt{k\rho c}, \quad (1)$$

where  $k$  is the thermal conductivity,  $\rho$  is the density and  $c$  is the specific heat of the material. The heat flux through the interface between two media is a function of the effusivity of each one. In case of defective material, if the defect effusivity is less than the material effusivity, the temperature on defective surface is higher than the defect

free zone temperature. Thus, by measuring the temperature distribution on the surface of the tested material, the defect presence can be revealed [9].

This paper presents a simple method for determining the thermal effusivity of defects. In the first part, we present the theoretical background of the proposed method. In the second part, we present and discuss the finite elements modeling results of a steel sample in the case of a defect with different nature (epoxy resin, thermal oil, wax (paraffin) and water). In the last, we validate the proposed method by experimental measurements.

## 2 Mathematical theory

The general equation of heat diffusion is given by:

$$\nabla[k\nabla T(r, t)] - \rho c_v \frac{\partial T(r, t)}{\partial t} = -w(r, t). \quad (2)$$

The one-dimensional solution of the Fourier equation for a dirac delta function in a semi-infinite isotropic solid is given by [10]:

$$T(t) = \frac{Q}{e_1\sqrt{\pi t}}. \quad (3)$$

When there is any subsurface defect or structure difference, the one dimensional solution is given by [11, 12]:

$$T(t) = \frac{Q}{e_1\sqrt{\pi t}} \left[ 1 + 2 \sum_{n=1}^{\infty} R^n \exp\left(\frac{-n^2 L^2}{\alpha t}\right) \right], \quad (4)$$

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where

- $Q$ : the input energy per unit area ( $\text{W}/\text{m}^2$ ),
- $e_1$ : the thermal effusivity  $\text{J}/(\text{K}\cdot\text{m}^2\cdot\text{s}^{1/2})$ ,
- $L$ : the defect depth (m),
- $\alpha$ : the thermal diffusivity ( $\text{m}^2/\text{s}$ ),
- $R$ : the thermal wave reflection coefficient between two media. It is expressed as [11]:

$$R = \frac{e_1 - e_2}{e_1 + e_2}, \quad (5)$$

- $e_2$ : the thermal effusivity of the defect.

For a defective material with effusivity  $e_1$ , the knowledge of the reflection coefficient  $R$  allows to determine the defect effusivity  $e_2$  by (6), derived from equation (5):

$$e_2 = e_1 \left( \frac{1 - R}{1 + R} \right). \quad (6)$$

We note that equation (4) can be expanded as:

$$T(t) = \frac{Q}{e_1 \sqrt{\pi t}} \left[ 1 + 2 \sum_{n=1}^N R^n \exp\left(\frac{-n^2 L^2}{\alpha t}\right) + 2 \sum_{n=N}^{\infty} R^n \exp\left(\frac{-n^2 L^2}{\alpha t}\right) \right], \quad (7)$$

where  $N$  is an integer greater than 1.

Let's consider the function  $f$  defined by:

$$f(t, R, N) = 2 \sum_{n=N}^{\infty} R^n \exp\left(\frac{-n^2 L^2}{\alpha t}\right). \quad (8)$$

We set  $\alpha$  value at the steel thermal diffusivity  $\alpha = 1.1934 \times 10^{-5}$  ( $\text{m}^2/\text{s}$ ) and the defect depth value at  $L = 1.1$  mm. Firstly, we look for the value of  $N$  which allows to have  $f(t, R, N) = 0$ , for this, we study the  $f$  variation as a function of time,  $R$  value and  $N$  value. Figure 1 represents the variation of  $f$  as a function of time for different values of  $N$ . The  $R$  value is taken equal to 1.

The obtained curves vary as exponential of time for  $N$  between 13 and 20. We note that the time interval where the function  $f$  becomes zero depends on the  $N$  value. The bearing where the function becomes zero increases with the value of the number  $N$ . Indeed, for  $N = 13$ ,  $f$  is zero on the interval time  $[0, 1.5]$ , whereas it is zero on the interval  $[0, 3.5]$  for  $N = 20$ .

We fix  $N$  equal to 30 and we study the  $f$  variation as a function of time for different values of  $R$  chosen arbitrarily (1, 0.9, 0.8, 0.7, 0.6 and 0.5). Figure 2 shows the obtained results.

The curves show functions become zero on intervals whose sizes depend on the  $R$  value. When  $R$  is increasing the interval width decreases. Indeed, for  $R = 0.5$ ,  $f$  is zero over the entire interval  $[0, 5]$ , whereas it is zero on the interval  $[0, 3.5]$  for  $R = 1$ .

In conclusion, the obtained functions become equal to zero on the interval time  $[0, 3.5]$  whatever the considered  $R$  value.

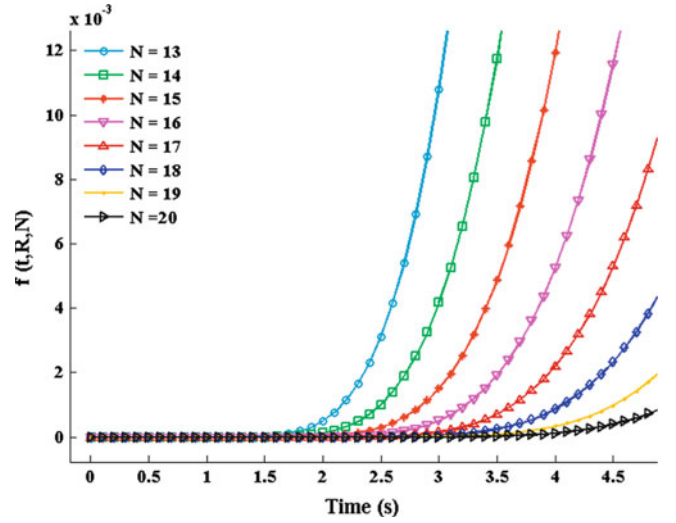


Fig. 1. Function  $f$  variation versus time for different values of  $N$  between 13 and 20 ( $R = 1$ ).

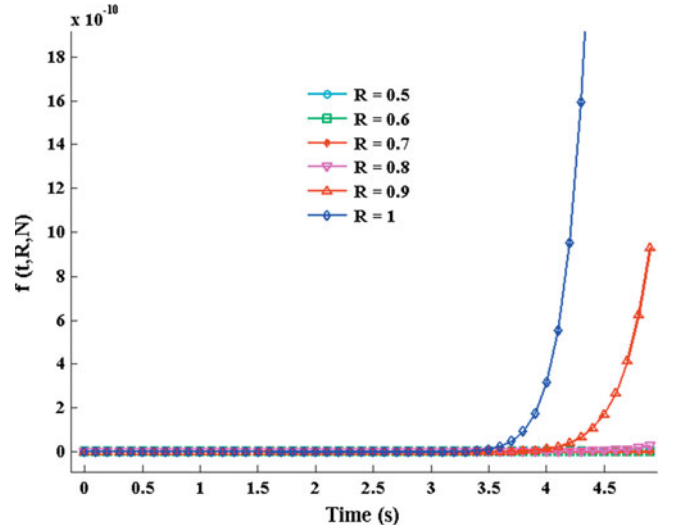


Fig. 2. Function  $f$  variation versus time for different values of  $R$  between 0.5 and 1 ( $N = 30$ ).

Let's consider  $N = 30$  and  $0 < t \leq 3.5$ . If the defect is a vacuum ( $R = 1$ ), the temperature evolution above the defect is written as:

$$T_{\text{vacuum}}(t) = \frac{Q}{e_1 \sqrt{\pi t}} \left[ 1 + 2 \sum_{n=1}^N \exp\left(\frac{-n^2 L^2}{\alpha t}\right) \right]. \quad (9)$$

If the defect thermal effusivity is unidentified ( $R$  unknown), the temperature evolution above the defect is written as:

$$T_{\text{defect}}(t) = \frac{Q}{e_1 \sqrt{\pi t}} \left[ 1 + 2 \sum_{n=1}^N R^n \exp\left(\frac{-n^2 L^2}{\alpha t}\right) \right]. \quad (10)$$

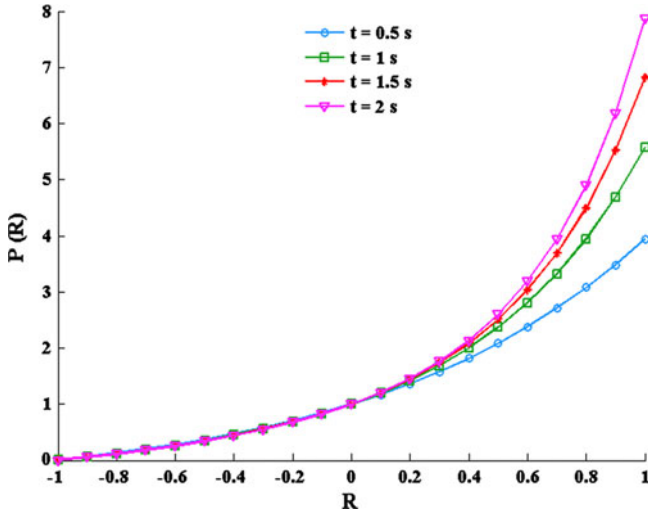


Fig. 3.  $P$  variation versus  $R$  value.

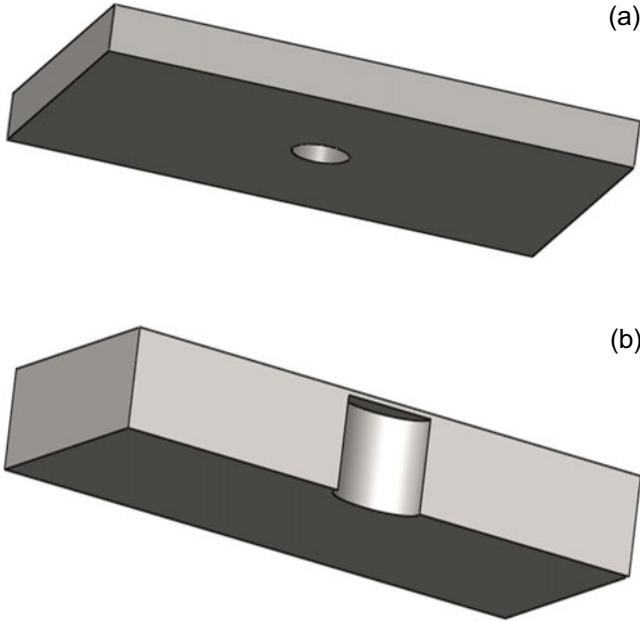


Fig. 4. (a) 3D steel sample with an inside defect. (b) Cutaway view of the steel sample with an inside defect.

From equations (9) and (10) we obtain:

$$\frac{T_{\text{defect}}(t)}{T_{\text{vacuum}}(t)} = \frac{\left[1 + 2 \sum_{n=1}^N R^n \exp\left(\frac{-n^2 L^2}{\alpha t}\right)\right]}{\left[1 + 2 \sum_{n=1}^N \exp\left(\frac{-n^2 L^2}{\alpha t}\right)\right]}. \quad (11)$$

Table 1. Thermal properties of the considered materials.

	Density kg/m <sup>3</sup>	Specific heat J/(kg K)	Thermal conductivity W/(m K)	Thermal effusivity J/(K m <sup>2</sup> s <sup>1/2</sup> )	Reflection coefficient $R$
Steel [13]	7850	475	44.5	12881.3576	–
Epoxy [14]	1275	1050	0.18	490.8920	0.9265
Oil [15]	886.2	1907	0.1891	565.3104	0.9159
Wax (paraffin) [16]	900	2900	0.24	791.4543	0.8842
Pure water [17]	997.13	4180	0.613	1598.4323	0.7792

For each  $t$  value  $0 < t \leq 3.5$ , the formula (11) can be written as:

$$B - 1 = \sum_{n=1}^N A_n R^n, \quad (12)$$

where

- $B = \frac{T_{\text{defect}}(t)}{T_{\text{vacuum}}(t)} \left[1 + 2 \sum_{n=1}^N \exp\left(\frac{-n^2 L^2}{\alpha t}\right)\right]$ ,
- $A_n = 2 \exp\left(\frac{-n^2 L^2}{\alpha t}\right)$ .

Equation (12) is a simple formula which allows to easily determine the coefficient  $R$ . It suffices to consider the polynomial  $P(R)$  defined by:

$$P(R) = \sum_{n=1}^N A_n R^n. \quad (13)$$

We represent in Figure 3 the variation of  $P$  as a function of  $R$  for  $t = 0.5$  s, 1 s, 1.5 s and 2 s.

We note that the function  $P(R)$  is strictly increasing on the interval  $[-1, 1]$ . So for each value of  $P(R)$  corresponds one value of  $R$ . Thus, for  $P(R) = B - 1$  corresponds one value of  $R$  which can be determined from the intersection of the straight line defined by  $y = B - 1$  and the curve of  $P(R)$ . The projection of this intersection on the axis  $R$  defined the desired value of  $R$ .

## 3 Numerical modeling

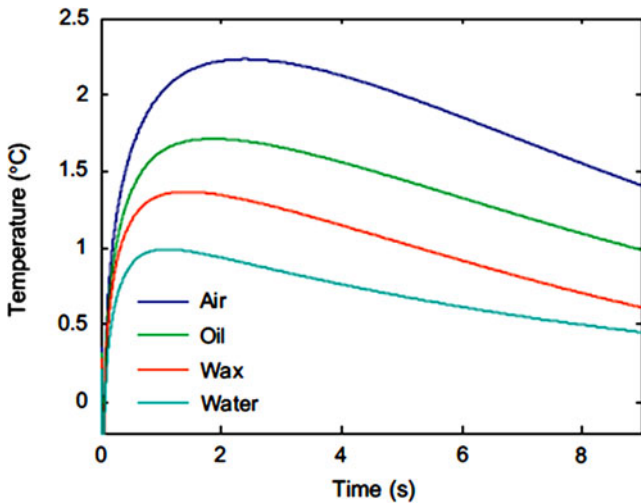
### 3.1 Numerical models

We consider five steel samples with dimensions 235 mm  $\times$  150 mm  $\times$  20 mm, each sample contains a defect of 25 mm in diameter and located at 1.1 mm from the heated surface (Figs. 4a and 4b). The associated defect for each sample is air, resin, oil, wax or water. For each sample, we applied a heat flux density equal to 6000 W/m<sup>2</sup> on the upper surface for 10 ms. The side faces are thermally insulated and the lower face is maintained at a constant temperature of 25 °C. The initial temperature of the samples is equal to 25 °C.

Numerical calculations of  $T_{\text{vacuum}}(t)$  and  $T_{\text{defect}}(t)$  were performed by using finite element software. A suitable program was developed in MATLAB for processing data.

**Table 2.** Measured defects thermal effusivity and measuring errors.

Measurement time	Defect	Reflection coefficient $R$	Calculated thermal effusivity $J/(K m^2 s^{1/2})$	Theoretical thermal effusivity $J/(K m^2 s^{1/2})$	Error (%)
$t = 0.5$ s	Epoxy	0.926	494.9224	490.8920	1
	Oil	0.916	564.7359	565.3104	0
	Wax (paraffin)	0.888	764.1483	791.4543	3
	Pure water	0.791	1503.2	1598.4323	6
$t = 1$ s	Epoxy	0.927	487.9809	490.8920	1
	Oil	0.917	557.7218	565.3104	1
	Wax (paraffin)	0.887	771.3797	791.4543	3
	Pure water	0.795	1471.1	1598.4323	8
$t = 1.5$ s	Epoxy	0.928	481.0466	490.8920	2
	Oil	0.919	543.7155	565.3104	4
	Wax (paraffin)	0.890	749.7086	791.4543	5
	Pure water	0.790	1511.2	1598.4323	5
$t = 2$ s	Epoxy	0.925	501.8711	490.8920	2
	Oil	0.915	571.7574	565.3104	1
	Wax (paraffin)	0.890	749.7086	791.4543	5
	Pure water	0.792	1495.2	1598.4323	6



**Fig. 5.** Experimental contrast-time curves for  $L = 1.1$  mm.

The Table 1 shows the thermal properties of the considered materials and the reflection coefficient  $R$  for each defect.

The determination of defects thermal effusivity will be made by applying the simple method presented in this manuscript. The obtained results are shown in Table 2.

### 3.2 Determination of defects thermal effusivity

The table shows the possibility of approaching, at an acceptable error, the defect thermal effusivity. Indeed, for most defects, the found thermal effusivity is very close to the actual value and the relative error remains less than 8%. So, the presented method can be important in the classification of defects in materials.

## 4 Experimental validation

To validate the effectiveness of the proposed method, we apply it at the experimental data shown in the Figure 5 as following [18].

The obtained results are shown in Table 3.

**Table 3.** Experimental measurements of defects thermal effusivity.

Measurement time	Defect	$R$ value determined by the new method	$R$ value in the literature [18]	Error (%)
$t = 0.5$ s	Oil	0.912	0.911	0
	Wax	0.817	0.833	2
	Water	0.671	0.667	1
$t = 1$ s	Oil	0.914	0.911	0
	Wax	0.814	0.833	2
	Water	0.672	0.667	1
$t = 1.5$ s	Oil	0.913	0.911	0
	Wax	0.819	0.833	2
	Water	0.670	0.667	0
$t = 2$ s	Oil	0.915	0.911	0
	Wax	0.822	0.833	1
	Water	0.675	0.667	1

Table 3 shows the effectiveness of our method in the experimental measurement of defects thermal effusivity. Indeed, at a relative error less than 3%, the measured  $R$  values are the same as those cited in literature. We can determine the defects thermal effusivity from the found  $R$  value. The proposed method in this work is of great importance for the characterization of defects in the material using pulsed thermography.

## 5 Conclusion

In this work, we presented a graphical simple method for determining the thermal effusivity of defects using data from pulsed thermography. We presented, in the first part, the theoretical background of the proposed method. Then we discussed the finite elements modeling results of a steel sample in the case of a defect with different nature (epoxy resin, thermal oil, wax (paraffin) and water). The results from simulations for thermal effusivity estimates using this method were verified by experiments using steel samples infected by cylindrical cavities filled in by different substances. The maximum error between the experimentally measured results and the FEM simulations was less than 3%. The obtained results have shown the ability of this method to characterize defects in a steel sample. So, the proposed method is of great importance in identification and characterization of defects in structures.

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