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Evaluation of shielding effectiveness of composite wall with a time domain discontinuous Galerkin method*

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Abstract. This article presents a time domain discontinuous Galerkin method applied for solving the conservative form of Maxwells' equations and computing the radiated fields in electromagnetic compatibility problems. The results obtained in homogeneous media for the transverse magnetic waves are validated in two cases. We compare our solution to an analytical solution of Maxwells' equations based on characteristic method. Our results on shielding effectiveness of a conducting wall are same as those obtained from analytical expression in frequency domain. The propagation in heterogeneous medium is explored. The shielding effectiveness of a composite wall partially filled by circular conductives inclusions is computed. The proposed results are in conformity with the classical predictive homogenization rules.

1 Introduction

Today, the development of new devices in electronic, electricity or in electromagnetics leads to an increasing importance for solving compatibility problems. The optimization techniques for immunity and emissivity are built through the modeling and characterization of used materials. Many discretization methods allow evaluating the shielding effectiveness of systems [1]. Unfortunately, the new materials encountered in applications are often heterogeneous and complex. This leads to increasing difficulties to implement methods such as finite element methods or finite differences methods. For example, when considering a medium with small-sized heterogeneities, the matrices generated by finite element are poorly conditioned and numerical resolution requires considerable efforts.

This article presents a discontinuous Galerkin discretization in time domain for solving the Maxwells' equations in materials and evaluating their shielding effectiveness. This method is a suitable tool for studying waves propagation in heterogeneous media because of its discontinuous aspect allowing to easily discretize objects of different sizes or shapes and provide a better consideration of discontinuous properties [2]. This particularity will allow us to study complex systems. These kind of methods are well adapted for parallel computing because the generated matrices are block diagonals. When they are combined with efficient numerical schemes such as Leap Frog or explicit multi-floors Runge-Kutta, their performances considerably increase [3].

In the following, we present outlines of the discretization of the conservative form of Maxwells' equations. Validation of this approach is performed on 2D cases in TMz and TEz polarizations. Our results of shielding effectiveness are compared to analytical expression in the case of homogeneous medium, and to the classical homogenization rules in the case of a wall partially filled with conducting inclusions. Note that our implentatation is under GMSH in order to benefit its user-friendly meshing, its parametrization tools and post-proceesing techniques [4].

2 System

Let E, D, H, B and J be, respectively, the electric field, the electric induction, the magnetic field, the magnetic induction and the current density. We consider a medium of permittivity ϵ and permeability μ. The constitutive laws are (1):

$$B = \mu H \quad \text{and} \quad D = \epsilon E. \tag{1}$$

The fields satisfy, the Maxwells' equations which constitute a system (2) of 6 unknowns, $E_{\{x,y,z\}}$ and $H_{\{x,y,z\}}$:

$$\begin{cases} \epsilon \dfrac{\partial E}{\partial t} - \text{rot} H = -J, \\ \mu \dfrac{\partial H}{\partial t} + \text{rot} E = 0. \end{cases} \tag{2}$$

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Let us note $u = \begin{pmatrix} E \\ H \end{pmatrix}$, $M = \begin{pmatrix} \epsilon & 0 \\ 0 & \mu \end{pmatrix}$ and $Q = \begin{pmatrix} -J \\ 0 \end{pmatrix}$. The system (2) takes a conservative form given by (3):

$$M\frac{\partial u}{\partial t} - \nabla \cdot F(u) = Q, \tag{3}$$

with, $F(u) = \begin{bmatrix} F_1(u) \\ F_2(u) \\ F_3(u) \end{bmatrix}$, where $F_i(u) = \begin{pmatrix} -e_i \times H \\ e_i \times E \end{pmatrix}$ and e_i is the unit vector $(i = 1, 2, 3)$.

2.1 Weak formulation and discretization

Let us consider a triangulation $\mathcal{I}_h = \cup\Omega_e$ of the domain Ω. The discontinuous Galerkin approach is dedicated for solving conservative forms of PDEs. It combines discretization tools of finite element (FE) and finite-volume (FV) methods and consists in solving the weak formulation of the system on each Ω_e. In our study, the weak formulation is written as (5):

$$\frac{d}{dt}\int_{\Omega_e} \epsilon E \cdot \phi_e \mathrm{d}\Omega_e + \int_{\Omega_e} (e_i \times H) \cdot \nabla\phi_e \mathrm{d}\Omega_e - \int_{\partial\Omega_e} (e_i \times H) \cdot n_{\Gamma_e}\phi_e \mathrm{d}\Gamma_e = -\int_{\Omega_e} J \cdot \phi_e \mathrm{d}\Omega_e, \tag{4}$$

$$\frac{d}{dt}\int_{\Omega_e} \mu H \cdot \phi_e \mathrm{d}\Omega_e - \int_{\Omega_e} (e_i \times E) \cdot \nabla\phi_e \mathrm{d}\Omega_e + \int_{\partial\Omega_e} (e_i \times E) \cdot n_{\Gamma_e}\phi_e \mathrm{d}\Gamma_e = 0, \tag{5}$$

where ϕ_e is a test function and $\Gamma_e = \partial\Omega_e$ is an interface between two elements of $\mathcal{I}_h$ or a part of $\partial\Omega$.

Rules for evaluating the different terms of the weak formulation exploit properties of mesh parametrization also called mapping. The mapping is based on a bijective function such as $\Phi(x, y, z) = (\xi, \eta, \zeta)$, which allows to transform the physical space (x, y, z) into a (reference) parametric space (ξ, η, ζ) [4]. The basis functions in the parametric space are the linear combinations of $\xi^r\eta^s\zeta^t$, where $s + r + t \leq p$. The terms of the weak formulation are evaluated in the parametric space, where derivation and integration operations are more convenient, and then mapped in the physical space.

2.2 FE discretization on each element

On each Ω_e a finite-element approximation space is defined. Its basis functions are Lagrange polynomials of degree p. The number of nodes on Ω_e is given by: $N = (p + 1)(p + 2)(p + 3)/6$. The discrete solution on Ω is defined by:

$$u^{\Omega_e} = \sum_{j=1}^{N} u_i^{\Omega_e}\phi_e. \tag{6}$$

The elementary mass matrices of permittivity and permeability are determined from L^2 scalar product:

$$M_{\alpha,\Omega_e} = \int_{\Omega_e} \alpha\phi_e \cdot \phi_e d\Omega_e, \tag{7}$$

where $\alpha = \{\epsilon, \mu\}$. The terms on Ω_e containing $(e_i \times E)$ and $(e_i \times H)$ in (5) depend on the elementary stiffness matrix given by:

$$S_{\Omega_e} = \int_{\Omega_e} \phi_e \nabla\phi_e d\Omega_e. \tag{8}$$

The resulting matrices of the system on Ω are block diagonal.

2.2.1 Flux term on interfaces

As in an FV method, interface terms on $\partial\Omega_e$ in (5) are treated by an expression F that characterizes exchanges between elements and which verifies $F^{\Omega_{e_1},\Omega_{e_2}} = -F^{\Omega_{e_2},\Omega_{e_1}}$, where Ω_{e_1} and Ω_{e_2} are neighboring. Its construction needs the following functions at the interface: the mean value

$$\{u\} = (u^{\Omega_{e_1}} + u^{\Omega_{e_2}})/2, \tag{9}$$

and the jump value

$$[\![u]\!] = (u^{\Omega_{e_1}} - u^{\Omega_{e_2}})/2. \tag{10}$$

Many expressions of F have been proposed for the Maxwells' equations [5]. We choose expression (11) based on seeking invariants at interfaces as in a Riemann solver techniques used in fluid mechanics [6]:

$$F = \begin{cases} n \times \dfrac{\{ZH\}}{\{Z\}} - \gamma n \times \dfrac{(n \times [\![E]\!])}{\{Z\}} \\ n \times \dfrac{\{YE\}}{\{Y\}} + \gamma n \times \dfrac{(n \times [\![H]\!])}{\{Y\}}, \end{cases} \tag{11}$$

where $Z = 1/Y = \sqrt{\mu\epsilon^{-1}}$, n is the outward normal at interface, and γ is a parameter such as $\gamma = 0$ for centered fluxes and $\gamma = 1$ for upwing fluxes.

Different numerical schemes are implemented in our code: Crank Nicolson and Leap Frog for $\gamma = 0$, implicit and explicit Runge-Kutta 44 for $\gamma = 1$.

2.3 Propagation in infinite medium

We consider two nonconducting media separated at $x = 0$. They are of same permeabilities and different permittivities. Initially, we impose an impulsion of TM_z polarization located in $x > 0$ and propagating toward $x < 0$, such as:

$$E_z(t = 0) = \begin{cases} \cos[k(x - x_0)] & \text{if} \quad x \in [x_0 - \lambda, x_0 + \lambda], \\ 0 \end{cases} \tag{12}$$

with, λ the wavelength and $k = 2\pi/\lambda$.

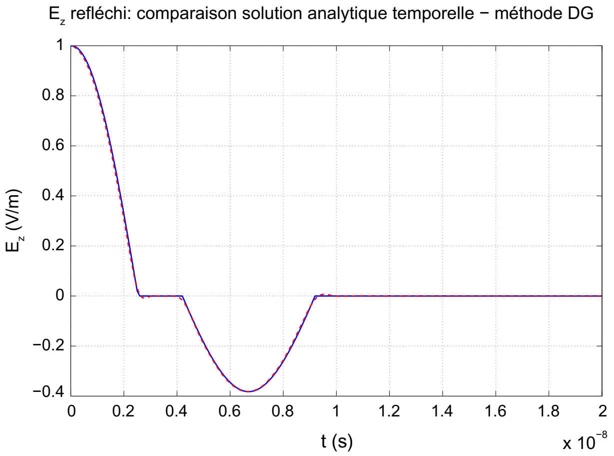

Fig. 1. Comparison of reflected E_z at $x = 1$ m.

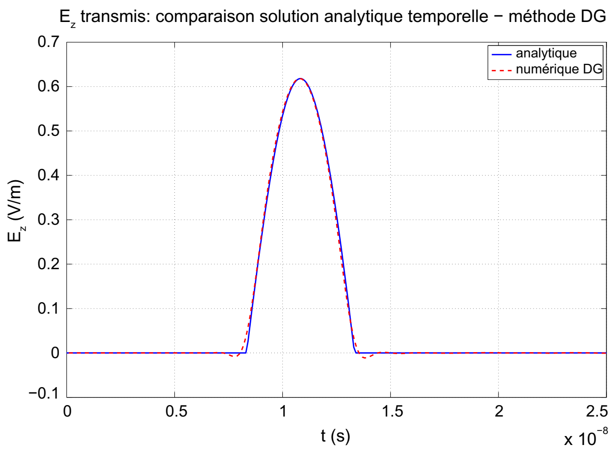

Fig. 2. Comparison of transmitted E_z at $x = -1$ m.

In this example, the geometry is a domain such as in $x \in [-3 \text{ m}, 3 \text{ m}]$. The unknowns of the problem are E_z, H_x and H_y and initial condition (12) is defined with $x_0 = 1$ m, $\lambda = 0.75$ m. The absorbing boundary conditions are set on left and right sides, whereas magnetic wall boundary conditions are imposed on top and bottom sides.

For relative values of permeabilities $\mu_+ = \mu_- = 1$ and permittivities $\epsilon_- = 5$ and $\epsilon_+ = 1$, we compare the reflected and transmitted electric field to those obtained from the analytical expression of Maxwells' equations solution based on method of characteristics seeking [7]. A good agreement is observed in Figure 1 for reflected field at $x = 1$ m and in Figure 2 for transmitted field at $x = -1$ m.

3 Shielding effectiveness

Let us consider a system subjected to an incident field E^i and let us note E^t the transmitted field. The shielding effectiveness of this system is defined by:

$$SE_{dB} = 20 \log \left| \frac{E^i}{E^t} \right|. \qquad (13)$$

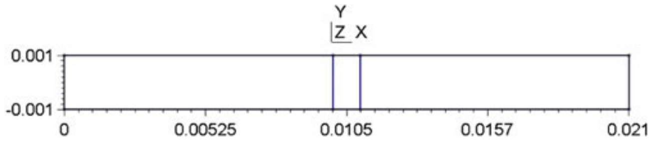

Fig. 3. Geometry of our simulation: Homogeneous wall of thickness $e = 1$ mm placed in vacuum.

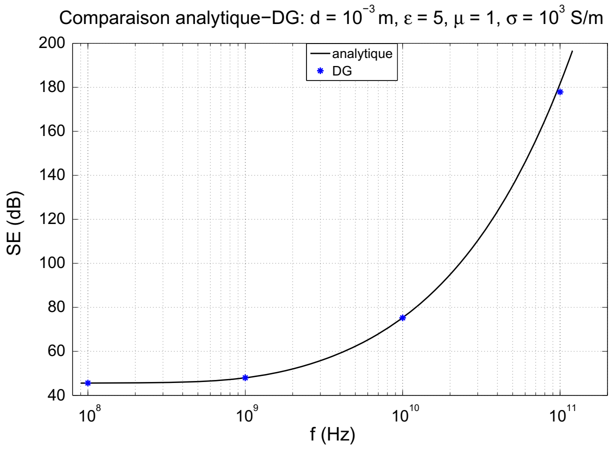

Fig. 4. Comparison of shielding effectiveness of homogeneous wall.

3.1 Homogeneous wall

We study an homogeneous wall of thickness e (Fig. 3) in vacuum, such as $J = \sigma_p E$, where σ_p is the conductivity of material. An absorbing boundary condition is imposed on the left side, whereas magnetic boundary wall conditions are set on top and bottom sides. An incident planar wave of TMz polarization is applied using a Dirichlet boundary condition on the right side and propagates toward the wall.

For the following characteristics: $e = 1$ mm, $\sigma_p = 10^3$ S/m, $\epsilon_p = 5$ and $\mu_p = 1$, we compute shielding effectiveness of the wall for $f \in \{0.1, 1, 10, 100\}$ GHz and compared our results to those obtained from analytical developments of Maxwell equations in frequency domain [8]. Our results coincide with analytical formula Figure 4.

3.2 Heterogeneous wall

The homogeneous wall of thickness $e = 1$ mm is replaced by a nonconducting medium of same thickness. We filled it with circular inclusions stored in square form Figure 5, of radius r_i, permittivity ϵ_i, permeability μ_i and conductivity σ_i. For the following characteristics of the wall, $\epsilon_p = 5$, $\mu_p = 1$, $\sigma_p = 0$ and inclusions, $\epsilon_i = 1$, $\mu_i = 1$, $\sigma_i = 10^3$ S/m, we determine shielding effectiveness in two cases of surfacic inclusions rate: (a): $\tau = 20\%$ and (b): $\tau = 76\%$.

For TEz polarization, electric wall boundary conditions are imposed on top and bottom sides and an absorbing boundary condition is set on left side. We evaluate the shielding effectiveness of (a): $\tau = 20\%$ for $f \in \{1, 5, 10, 11\}$ GHz when material is subjected to an incident planar wave $H_z(t)$. Our results shown in Figure 6

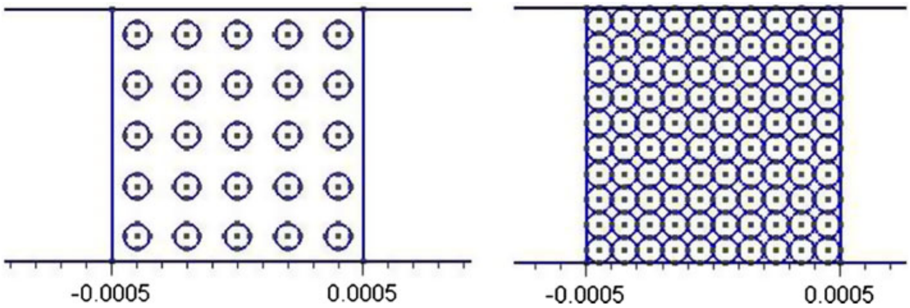

Fig. 5. Wall of thickness $e = 1$ mm partially filled with inclusions of radius $r_i = 0.05$ mm: (left) $\tau = 20\%$ (right) $\tau = 76\%$.

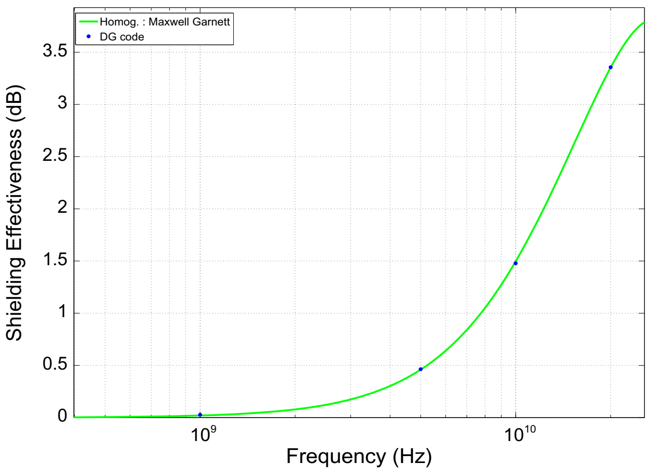

Fig. 6. TEz polarization: comparison of shielding effectiveness of a wall filled with 20% of inclusions.

are in conformity with Maxwell-Garnet homogenization approach [9].

For TMz polarization, magnetic wall boundary conditions are imposed on top and bottom sides and an absorbing boundary condition is set on left side. When an incident planar wave $E_z(t)$ of frequency $f = 10$ GHz is considered, we determine shielding effectiveness of (a): $\tau = 20\%$ and (b): $\tau = 76\%$ are determined. The values $SE_{\text{dB}}^{20\%} = 38$ dB and $SE_{\text{dB}}^{76\%} = 66$ dB are the same as those obtained from homogeneous wall whose conductivity is $\sigma_{\text{eq}} = \tau\sigma_i$. We obtain the homogenization result which states that equivalent conductivity is proportional to surfacic inclusions rate [10].

4 Conclusion

This paper presents a discontinuous Galerkin method and its validation on evaluating shielding effectiveness of materials. We highlight the fact that this discontinuous approach is appropriate for studying complex systems. An illustration example is proposed on computing shielding effectiveness of an heterogeneous periodic wall. Our results are in accordance with homogenization laws. That shows robustness of this method and justifies its implementation for describing waves propagation in complex materials.

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