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Improved AC current measurement approach in multiphase cable using proper orthogonal decomposition*

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Abstract. Accurate monitoring of energy consumption is key for electrical energy management in industry. This implies the use of low-cost and easy-to-install measuring chains that can be placed directly around three-phase cables without interrupting power delivery. This paper describes an innovative current measurement method using a magnetic sensor array dispatched around three-conductor cables. Analytical solution is given for the measurement problem and results from simulations.

1 Introduction

A key step in energy management in industry, and one of the necessary conditions for sustainability of implemented solutions, is the ability to measure, monitor and control this energy. Upstream of any approach to energy saving is the energy diagnosis: in order to identify ways to optimize energy consumption, one must have a clear view of energy flows in the system. As much as electrical appliances are concerned, the current measurement in a multiphase system is necessary for evaluating the energy consumption in power systems. In most instruments, current measurements are done by measuring the magnetic field radiated by the conductors, either by basic principles (transformers, Rogowski coils) or by more sophisticated techniques using independent measurements of the magnetic induction field. Of course, many well-established technologies do exist for measuring the current flowing in a single conductor but, at our best knowledge, no method currently exists for non-intrusive (i.e., that does not require the opening of the electrical power circuit to install the sensor, or having access to each single conductors) current measurement in a multiconductor system, the configuration of which is not necessarily known. D'Antona et al. propose a new principle for simultaneous measurement of polyphase currents flowing in parallel conductors. These are copper bars of known configuration where satisfactory experimental results are presented [1]. These results were extended to take into account the electromagnetic interference [2,3]. Detailed analyses of several non-intrusive technologies are given in [4–6]. All these methods are based on the assumption of knowing the position of the conductors. In [7] a current sensor dedicated to an electrical device of known geometry is presented with Fluxgate magnetometers type [8,9] have shown very good results for the current measurement, but for a single conductor. Finally, sensors based on the technique of measuring the magnetic field by optical fiber with very good accuracy (0.1% to 1%) are presented in [9–11]: the disadvantages of sensors are the fact that they are designed for a single conductor and their high cost due to the use of crystals.

The objective of this work is to design and test an innovative low-cost measuring device to monitor and analyze energy consumption. The measurement must be possible outside of electric cabinets, by placing the power meter directly around three-phase cables, without interrupting power supply. This innovative current sensor for multiphase cable using magnetic field measurements that is detailed in this paper was also patented [12].

The principle of the device depicted in Figure 1 is to measure the magnetic field distribution around a three-phase cable, by means of an array of sensors, placed in proximity of the cable, and to reconstruct the currents in each conductor. This process involves the solving of inverse problems, with additional difficulties related to the unknown characteristics of the cable: its internal structure (angular position of the conductors, conductors radii ...) and its actual position (rotation, decentering ...) with respect to the measurement device. In this paper, we will focus only on the measurement of the magnetic field and on the reconstruction of the currents. The system is

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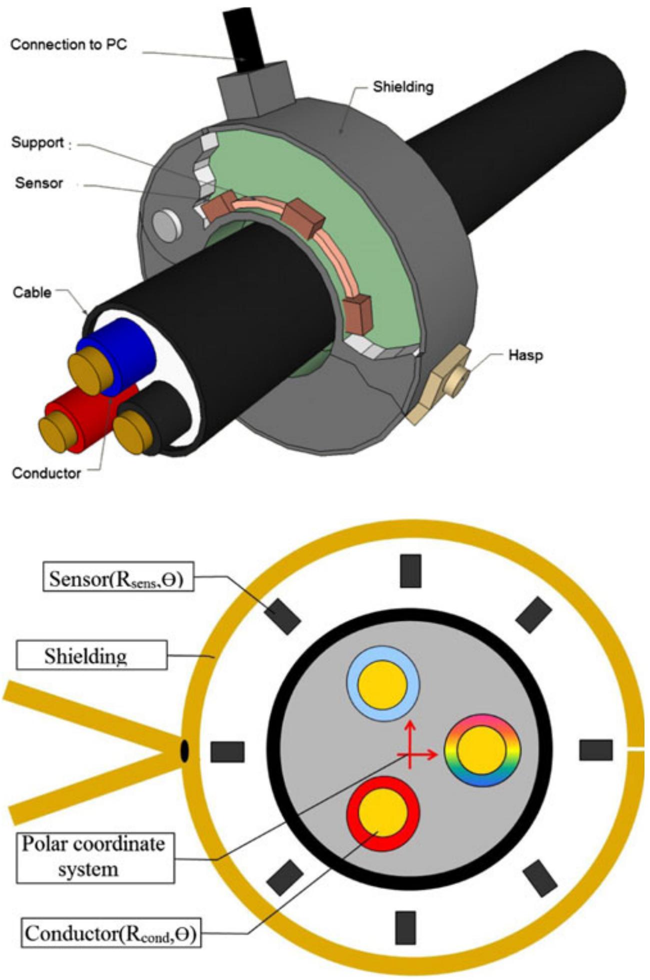


Fig. 1. Sketch of the current measuring device (a) and cross section (b).

composed of M-conductor cables, surrounded by N magnetic sensors, dispatched on a circle of radius R_{sens} (Fig. 1). Each conductor is modeled as an infinitely long straight wire, dispatched at equal angles on a centered circle of radius R_{cond}. Let θ be the angle between the first wire and the x (i.e., horizontal) axis. A magnetic shielding could be used to avoid electromagnetic perturbation.

2 Methodology of reconstruction

The well-known Biot-Savart law (1) allows computing the flux density $\mathbf{b}$ radiated by a current i flowing in an infinitely long straight cable, provided that magnetic perturbations (i.e., no shielding and/or any other magnetic material) are negligible in proximity of the cable:

$$\mathbf{b} = \frac{\mu_0 i}{2\pi r}, \tag{1}$$

where r is the distance between each conductor and sensor.

Equation (1) is linear with respect of i, therefore the flux densities $\{\mathbf{b}_i\}$ measured by the magnetic sensors can be linked to the currents $\{i_j\}$ flowing in the conductors by:

$$\mathbf{B} = \begin{pmatrix} K_{11} & \cdots & K_{1M} \\ \vdots & \ddots & \vdots \\ K_{N1} & \cdots & K_{NM} \end{pmatrix} \times \mathbf{I}, \tag{2}$$

where the vectors $\mathbf{B}$ and $\mathbf{I}$ collect respectively the components if the measured flux density $\mathbf{b}_i$ and the currents i_j, and the coefficients K_{ij} depend only on the geometry of the system (through the parameters R_{cond}, R_{sens} and θ) and can be determined analytically by (1) for different positions of sensors and conductors. If a magnetic shielding is present, these coefficients cannot be anymore computed by (1), but the linear relation (2) still holds, provided that no saturation is present, and that the magnetic material can be approximately assumed to be linear: this is likely to be the case, due to the low magnitude of the radiated magnetic field. Proximity effects and disturbances from conductors external to the cable are neglected. We assume that the number M of conductors in the cable is known. In general the number N of sensors will be bigger than the number of cables, so that the matrix $\mathbf{K}$ of coefficients K_{ij} will not be square. As a consequence, an estimate $\hat{\mathbf{I}}$ of the currents $\mathbf{I}$ flowing in the conductors can be computed by solving (2) in the sense of least squares:

$$\hat{\mathbf{I}} = \mathbf{K}^{+}\mathbf{B}, \tag{3}$$

where $\mathbf{K}^{+}$ is the Penroses pseudo-inverse [13] of $\mathbf{K}$. In practice the result will depend on two important parameters: the position of conductors and disturbances. These two points are discussed hereafter.

2.1 Determination of the positions of conductors

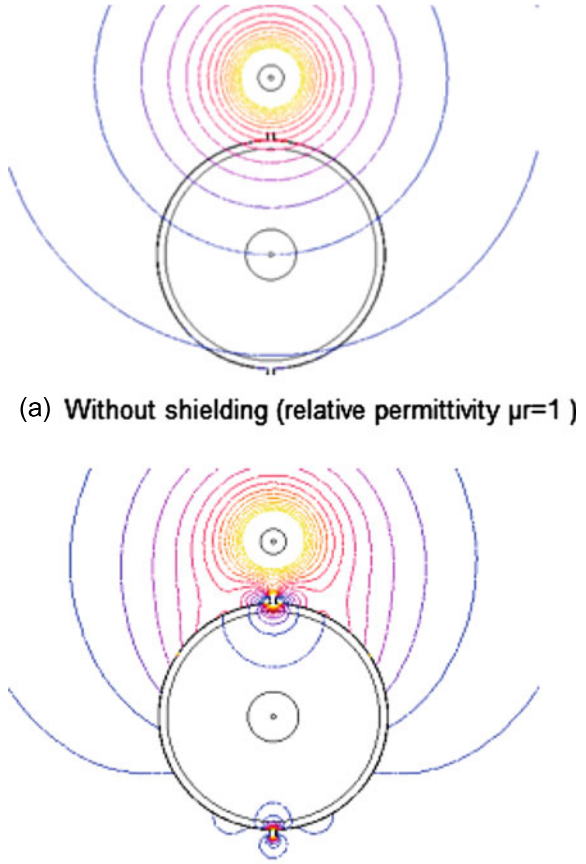
In order to compute the estimate (3) the real positions of the conductors (i.e., the parameters R_{cond} and θ) have to be determined. In order to determine the real conductor positions, we minimize the norm of the residual vector $\mathbf{R}$:

$$\mathbf{R} = (\mathbf{KK}^{+} - \mathbf{Identity})\mathbf{B}. \tag{4}$$

In the ideal case (no disturbances, perfect adequacy of the model to the real configuration of the cable), the residual vanishes when $\mathbf{K}$ corresponds to the real position of the conductors. This minimization problem may be solved by exhaustive search through a large but limited number of possible configurations of the conductors. Note that this operation has to be performed only once (ideally during the installation of the sensor). Moreover, the computational cost for evaluating the objective function is very low, and only two parameters (R_{cond} and θ) have to be identified: thus, the overall computational cost of this problem is quite acceptable, even by using an exhaustive method (less than 1 s on a standard PC).

2.2 Disturbance rejection by using a magnetic shielding

In practice, the measurement may be disturbed by the proximity of other cables and/or electrical appliances. In order to make the measurement device less sensitive to such a disturbance, we designed a magnetic shielding placed around the cable. The finite elements (FE) software FLUX [14] and GETDP [15] have been used to evaluate the magnetic flux density radiated by an external cable with and without the shield in a representative case (Fig. 2). In this and in all the following computations the magnetic shield is a cylindrical sheet (radius = 45 mm, 1 mm thick) of magnetic material ($\mu_r = 1000$, conductivity $\sigma = 10^6$ S/m). In this example the external conductor is located at 70 mm from the center of the multiconductor cable. The magnetic shielding reduces the influence of the external cable on the measured flux density, and therefore makes the measurement device less sensitive to external disturbances. However, the matrix **K** cannot be anymore computed analytically by (1), therefore we used the Finite Element GETDP code (2D magnetodynamic formulation). As a FE code cannot be decently implemented in embedded micro-controllers, an interpolation table is computed off-line with the values of the flux density computed for one conductor located at the different position of a 2D grid. This table – which can be easily uploaded in a micro-controller – is then used to compute the matrix **K** for any configuration of the conductors in the cable. This method allows to reconstruct the currents in the conductors, despite the distortion in the magnetic field caused by the shielding. In the results presented hereafter, the computational grid used for building the table has a resolution of 1 mm × 1°, which requires solving 1200 FE problems.



(a) Without shielding (relative permittivity μr=1)

(b) With shielding (relative permittivity μr=1000)

Fig. 2. Flux lines without shielding (a) and with a shielding (b).

2.3 Disturbance rejection by POD

Additional provision must be taken if the shielding is not sufficient to completely cancel out the influence of an external disturbing conductor. This can occur notably if the air gaps cannot be kept small enough, or if a power cable is located in close proximity to the multiconductor cable where the measurement is being performed. In order to overcome this problem, we developed a complementary disturbance rejection method, which consists in filtering out the signal coming from an external perturbation: to achieve this aim, it is necessary to obtain a magnetic signature of the external disturbance. In order to obtain such a signature, we performed a high number (1000) of simulations, in which a straight perturbing wire has been located in different positions (outside the area delimited by the magnetic shied) around the multiconductor cable. The values of the flux density computed in the positions of the magnetic sensors have been collected in the rows of a matrix **P**. Then we performed a proper orthogonal decomposition (POD – also called principal component analysis (PCA), or Karhunen-Loève transform) by computing the singular value decomposition (SVD) of **P**:

$$\mathbf{P} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{t}}.$$

A magnetic signature of the external disturbance can be obtained by taking a few number of columns of the matrix **U** corresponding to the higher singular values: the selected columns form a reduced basis which is used for rejecting any external disturbance. Hence, we build an extended system which is solved instead of (2) in order to reject external disturbances:

$$\mathbf{B} = \begin{bmatrix} \mathbf{K} \; \mathbf{K_p} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \xi \end{bmatrix}, \tag{5}$$

where the columns of $\mathbf{K_p}$ contain the reduced basis obtained by the POD, and the (unknown) vector ξ has no immediate physical meaning.

3 Numerical results

We developed a simulation tool by using MATLAB in order to simulate the behavior of the measurement device in several conditions. First, the interpolation matrix is computed off-line. Then the measurement of the flux density is simulated; a noise can be added to the simulated measurement. The simulated measurements are used to test the current reconstruction methods with and without the magnetic shielding, and with several reduced basis for disturbance rejection. This test is composed of two steps: first the position of the conductors in the cable is determined (device calibration); this position is then used for computing the estimate of the currents by (3) or (5).

3.1 Noised measurement without external disturbance

As first example, we simulated the measurement of currents flowing in a three-phase cable ($M = 3$) loaded with a balanceed charge of $i_{\max} = 50$ A ($i_{\text{eff}} = 35.4$ A) on each conductor, using $N = 8$ magnetic sensors. As expected, in the absence of any noise and external disturbance the result is perfect (up to the numerical precision). We simulated the effect of a additive white noise on the flux density measurement by adding a random Gaussian signal. Only the tangential component of the flux density is measured. Even for a rather low signal/noise (S/N) ratio of 3.3, the method provided a fairly good localization of the conductors and fairly good results for currents reconstruction (Tab. 1). The norm of the residual (4) is plotted in Figure 3 as a function of R_{cond} and θ, together with the exact position of the conductors and the estimated position (due to the symmetry of the system, only the range $\theta \in [0, 120°]$ is plotted): it can be observed that the estimated position is close to the true position.

3.2 Noised measurement with external disturbance

As a second example, we added to the previous configuration an external conductor, with a current of 1000 A at 150 Hz (this frequency has been chosen only in order to simplify the visual analysis of the results – the method is programmed in the time domain). We observe that even in the presence of a magnetic shielding, the estimated currents are very inaccurate if the standard equation (3) is solved (Tab. 2). Moreover, the measurement noise does not seem to be responsible for this degradation of performances: in that even without the measurement noise, the result is not much better. Conversely, if the first two vectors obtained by the POD are used as reduced basis together with (5) a much better estimate is obtained. This is confirmed both, by visual observation of the current in time domain (Fig. 4), and from the determined positions of the conductors, which are much more accurate in the latter case (not shown). In particular, when the reduced basis obtained from the POD and (5) are used (Fig. 4) the 150 Hz component (characteristic of the simulated external disturbance) is filtered from the estimated current; conversely, it can be clearly observed by a visual analysis when (3) is used.

When the S/N ratio decreases or when a higher number of vectors are used for the reduced basis, it can be observed that the advantage of using the POD (5) for rejecting external disturbances over the more classical (3) vanishes. This can be explained by the fact that the extended matrix of the linear system (5) generally has a worse condition number κ with respect to (3): for a S/N ratio = 10 we computed a condition number: $\kappa = 63.7$ for the standard matrix $\mathbf{K}$, $\kappa = 572$ for the matrix extended with a reduced basis of two vectors, and even $\kappa = 3440$ when four vectors are used. In this last case, the estimation obtained by solving (5) is much worse than that obtained by using the classical (3) if only $N = 8$ magnetic sensors are used, and we observe that increasing the number of sensors does not lead to a much improved accuracy (not shown).

Table 1. Efficacy currents measured without external disturbance (unit = A).

	$i_{\text{eff},1}$	$i_{\text{eff},2}$	$i_{\text{eff},3}$
(Exact)	35.4	35.4	35.4
S/N = 10	36.1	35.8	35.8
S/N = 3.3	36	38.9	39.2

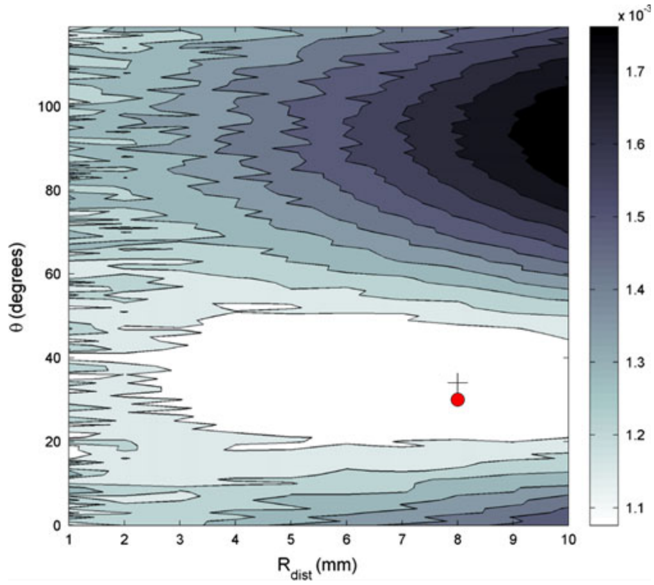


Fig. 3. Norm of the residual $||\mathbf{R}||$ as a function of R_{cond} and θ (• = true position, + = estimated position).

Table 2. Efficacy currents measured with external disturbance (unit = A).

	$i_{\text{eff},1}$	$i_{\text{eff},2}$	$i_{\text{eff},3}$
(Exact)	35.4	35.4	35.4
No noise	55.0	54.0	48.1
S/N = 3.3	55.3	54.5	48.7
S/N = 3.3, POD	31.5	29.0	32.1
S/N = 10	43.2	48.4	48.3
S/N = 10, POD	48.5	47.4	43.8

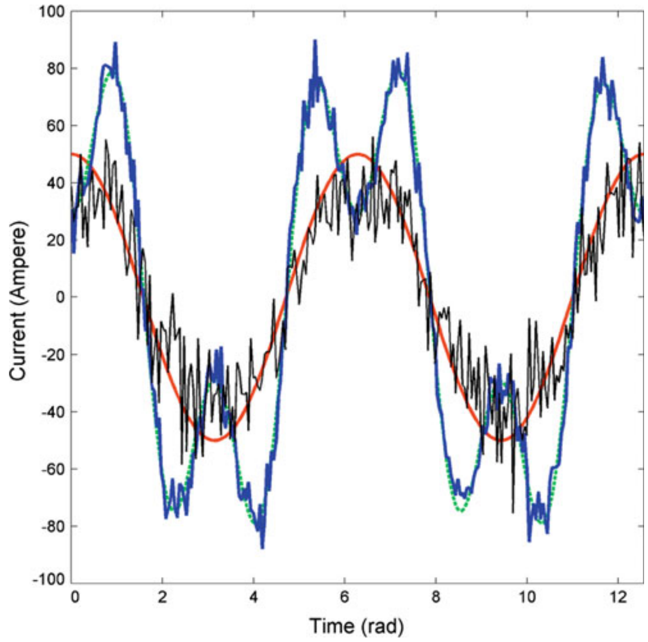


Fig. 4. Current in the first conductor of the cable: true current (red line); current estimated by (3) without noise (green line) and with a noise with S/N ratio = 10 (blue line); current estimated by (5) (black line).

4 Conclusion

A current reconstruction method for multiphase cables is presented in this paper. This method is based on magnetic sensors that are dispatched around the cable, and it allows to obtain an estimate of the currents flowing in each conductor of the cable despite the a priori unknown position of the conductors. Obtained results are fairly good, provided that the S/N ratio is high enough. The advantage of this measurement device is that it can be installed without powering down the electrical system: during installation, the position of the conductors is determined by solving an inverse problem, and the device is able to estimate in real time the current through each conductor. The problem of the presence of other conductors (external to the cable) which would disturb the measurements environment is solved by using a magnetic shielding, and by rejecting the signal by using a method based on POD. However, this last point still deserves further investigations, because indeed it may lead to worse estimate of the currents if the S/N ratio for the measurement of the flux density is too low. Moreover, the condition number of the matrix of the linear system gets higher when such a reduced basis is used: therefore, it is problematic to know in advance whether the extended or the classical matrix (i.e., Eqs. (5) or (3)) should be preferred.

Even if this study suggests that the proposed method could be interesting in a practical application, some points deserve more attention: in particular, the model used is probably oversimplified: a 3D modeling could allow to obtain more insight into the effectiveness of the shielding (leaking magnetic fields cannot be easily accounted by 2D computations). Second, the influence of incorrect installation (bad alignment with the cable) and of variations and/or tolerances should be quantified (parameters of the magnetic sensors, effect of aging ...). An even further step is to estimate the electric voltage of conductors – or at least the phase shift between currents and voltages – in order to calculate the electrical power consumption.

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