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Phase spectrum increments of the speckle pattern

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Abstract. It is well known that the interactions between a coherent light and a scattering medium produce a speckle pattern. But, even if intensity and phase statistics have been already well described for a long time, the statistics of the increments of the phase spectrum have never been explored. In this paper, we firstly introduce these concepts before demonstrate that phase spectrum increments contain informations on the medium observed. Moreover, we show that phase spectrum increments allow reaching informations on the media which are not present in the classical approaches based on the intensity. Monte Carlo simulations are used to this aim.

1 Introduction

Speckle, which is an interference phenomenon, appears when a spatially coherent light interacts with a rough surface or propagates itself in a random medium. Such a phenomenon can be observed in many physical applications, e.g., microscopic images, stellar images and others. There are two approaches of taking speckle into consideration. In one approach, speckle can be considered as a noisy phenomenon that pollutes observations. Hence the objective is suppress it, as far as practicable. In another approach, one can consider that it contains a lot of information about the system under observation. The choice between these two point of views depends on the specific application, i.e., in our case, bio-medical imagery. A systematic study of the statistical properties of the speckle was undertaken to investigate the classification of speckle in the context of particular dermatological pathologies. The temporal and spatial characteristics of speckle pattern depend on many factors; some of these are experimental factors, e.g., the wavelength, spectral bandwidth, or polarization of the illuminating radiation. On the other hand, the characteristics of the speckle will also depend on some physical factors which are linked to the properties of the observed media, e.g., texture, roughness, material, shape and so on. As speckle is an interference and diffraction phenomenon, the frequency based approach to study its behavior, through Fourier Optics, is frequently undertaken [1–3]. Goldfischer [2] first investigated the statistical properties of the speckle employing the power spectral-density and its autocorrelation function. The first and the second order statistics of the speckle [4] allow many applications in imagery [5]. Many researchers have also explored the relationships between the speckle dimensions and the experimental conditions [6–8]. More recently, the utility of the third-order intensity correlations have been investigated for several different applications. For example, Genack and Drake [9] developed an expression for the correlation of two speckle fields at different frequencies and Webster [10] successfully employed the third-order correlation to obtain the temporal response of a random medium.

In what concerns us, we demonstrated in previous papers [11,12] that speckle could be analyzed by the theory of fractals. From this point, we analyzed the consequences of this model. That led us to explore the statistics of the increments of the phase spectrum. So, after a brief recall of the statistics of the speckle in Section 2, we describe our approach in Section 3 before present our first results of simulation in Section 4. Few words of conclusion constitute the last Section 5.

2 The statistics of the speckle

2.1 Amplitude, phases and intensity statistics

Let us consider a monochromatic electric field of frequency ν_0, which is characterized by the equation (1):

$$U(x,y,z,t) = A(x,y,z)\exp(j2\pi\nu_0 t). \qquad (1)$$

In the first order statistics, the amplitude at a point in the space consists of the sum of all the out of phase contributions from many points of the scattering array. Hence, the equations (2.1) follow:

$$A(x,y,z) = \frac{1}{\sqrt{N}}\Sigma a_k(x,y,z) = \frac{1}{\sqrt{N}}\Sigma\,|a_k|\exp j\varphi_k, \qquad (2)$$

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where $0 \leq k \leq N$, considering the N points of scattering contribution. The consequence is that the amplitude may be considered as a random walk in the complex plane. Moreover, considering the theoretical hypotheses:

1. Amplitude $\frac{|a_k|}{\sqrt{N}}$ and phasis φ_k of the kth contribution are statistically independent and statistically independent with respect to the others too.
2. Phases φ_k of the contributions are uniformaly distributed on $[0; 2\pi]$.

Thus, from these hypothesis, Goodman [4] enunciated the probability density function of the intensity in equation (3).

$$P(I) = \frac{I}{2\sigma^2} \exp\left(-\frac{I}{2\sigma^2}\right). \quad (3)$$

Note that the probability density function of the phase is uniform and equal to $\frac{1}{2\pi}$.

3 The increments of the phase spectrum and their modelisation

3.1 Definition of the increments of the phase spectrum

Taking into account the principle of superposition of N wavelets, propagating in paraxial Fresnel approximation along the z axis and originated by N distinguishable sources with the presence of diffraction phenomena, their sum is mathematically given by the discrete Fourier transformation $FT(k_x, k_y)$ of the initial field distribution. Thus, if the diffraction patter corresponds to the FT of the field distribution multiplied for a scalar factor, the speckle pattern can be represented in terms of the inverse discrete Fourier transform as follows:

$$f(n_x, n_y) = \frac{1}{N_x N_y} \sum F(k_x, k_y) \exp j2\pi \left(\frac{k_x n_x}{N_x} + \frac{k_y n_y}{N_y}\right), \quad (4)$$

with $0 \leq n_x$, $k_x < N_x$, $0 \leq n_y$, $k_y < N_y$. $F(k_x, k_y)$ is the discrete Fourier transform at frequencies (k_x, k_y), n_x, n_y the spatial discrete coordinate where the pattern is calculed and N is the number of the N distinguishable wavelt sources. The Fourier transform is a complex number, which can be characterized by its amplitude and phase:

$$F(k_x, k_y) = a(k_x, k_y) \exp(j\theta(k_x, k_y)), \quad (5)$$

here, $a(k_x, k_y)$ represents the Fourier amplitude corresponding to (k_x, k_y) and θ is the phase angle. While the Fourier amplitude has been studied extensively, the phase spectrum has only received a limited attention. In fact, since there is not any privileged scattering direction in turbid media, the final phase spectrum is uniform and the phase is a random variable. As consequence the phase gradient will simply be the difference between two uniformly-distributed random variables and will itself be uniformly distributed, according to the algebra of random variable, between $-pi$ and pi. Whereas, the paraxial Fresnel approximation of the propagation respect the z axis is considered. Then we introduced a quantity $D_{\overrightarrow{\delta}}(\overrightarrow{k})$, defined by:

$$D_{\overrightarrow{\delta}}(\overrightarrow{k}) = \theta(\overrightarrow{k} + \overrightarrow{\delta}) - \theta(\overrightarrow{k}), \quad (6)$$

we refer to $D_{\overrightarrow{\delta}}$ as the phase gradient for small $\overrightarrow{\delta}$, i.e., $D_{\overrightarrow{\delta}} \simeq \overrightarrow{\nabla_{\overrightarrow{k}}}\theta.\overrightarrow{\delta}$, with $\overrightarrow{k} = (k_x, k_y)$. Here, we simply calculate gradients in the $(\overrightarrow{x})$ and $(\overrightarrow{y})$ direction of the speckle patterns independently, whereas there is not any preferred scattering direction or linked phenomena among the two ones. Since the difference between two circular random variables is itself a circular random variable, the distribution of $D_{\overrightarrow{\delta}}(\overrightarrow{k})$ should be uniform for each direction x and y.

3.2 Circular statistics of variates

In probability theory and statistics, the Von Mises distribution is a continuous probability distribution [13]. It can be seen as the circular version of the so called "gaussian" distribution, since it describes a continuous distribution of a variate with period 2π. It is used in application of circular statistics where a distribution of angles are found to be the result of the addition of many small independent angular deviations, as our case. Note that, in the particular case of the increments of phase of a speckle and from a theoretical and physical point of view, it is not yet possible to explain why the Von Mises distribution appears to be the more convenient for this non linear regression. The general form of the Von Mises probability density function (PDF) for the angle θ is given by:

$$p_0(\theta \mid \mu, \kappa) = \frac{1}{2\pi I_O(\kappa)} \exp\left(-\kappa \cos(\theta - \mu)\right), \quad (7)$$

where $I_0(\kappa)$ is the modified Bessel function of zero order. The parameters μ and κ are the location parameter and the concentration (or shape) parameter respectively. As κ approaches 0, the Von Mises PDF approaches a uniform density function $p_{0[-\pi,\pi]}(\theta) = \frac{1}{2\pi}$. In the case where κ is large, the Von Mises PDF become very concentrated about the angle μ with κ being a measure of the concentration, and for a large enough κ, the distribution becomes a normal distribution in θ with mean μ and variance $1/\kappa$, i.e., $N(\mu, 1/\kappa)$. The trend of Von Mises PDF is illustrated in Figure 1.

4 Results

Like enounced in introduction, many experimental factors can modify the speckle pattern and its statistics. From this simple observation, it appeared to us more adapted to use a simulation of the Monte Carlo type in order to show than the increments of the spectrum of phase were related to the physical properties of the medium observed. To this aim, we used the so called "EMC simulation" published

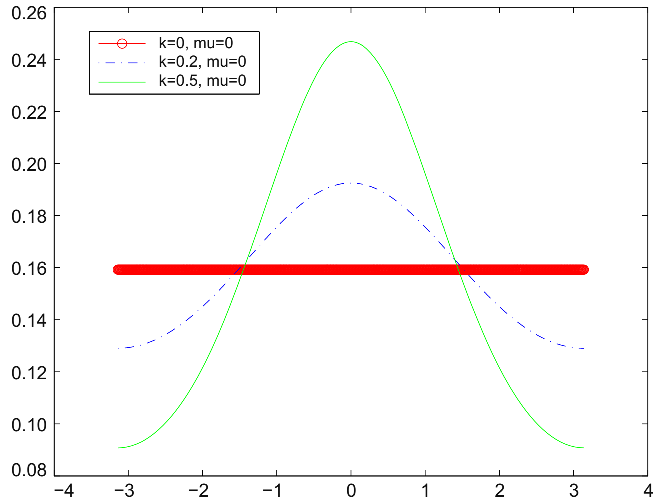

Fig. 1. Von Mises distribution for different values of κ.

by Min Xu in 2004 [14]. This code allows the simulation of the plane wave backscattering from an aqueous solution of polystyrene sphere inside a slab. In all our simulation, we used an unpolarized incident light which is $\lambda = 632.8$ nm and considering a propagation normal to the surface of the slab and which thickness is $L_z = 41_s$. A total of 20×10^7 photons are launched into the medium. On the other side, concerning the sample medium, the size of the polystyrene sphere is $d = 0.46\,\mu$m. It allows us to define the so called "size parameter" of the Mie sphere as $\pi d n_{\text{water}}/\lambda$, which is a classical parameter in Monte Carlo simulation.

Note that like mentioned by Min Xu [14] in his paper, "EMC" only probes an instantaneous picture of the disordered medium contrary to the ensemble average realized in experiments. But, "EMC" record both the electric field (instantaneous picture) and the Stokes vectors (ensemble average) in simulation. To obtain a speckle pattern which verify statistics of intensity and phase illustrated by the first part of this paper, it is indispensable to normalize the electric field by the mean of the element of the scattering amplitude matrix. To illustrate this point, we can compare Figures 2c and 3c. It is clear that considering the non normalized speckle, the statistics of the phase do not respect the uniformity. Experimentally, it is theoretically possible to obtain this speckle pattern by a judicious choice of the source and the camera in order to avoid the average due to the movements of the scatterers for example. Considering the simulation, the camera is in contact with the medium.

Thereafter, we keep the same parameters than detailed previously, but we gradually increase the refractive index of the sphere from 1.35 to 1.85.

4.1 Variation of the refractive index

In our view, it is really important to well understand what happens to the scattering properties of this medium when the refractive index of the sphere increase. In a first time, we can refer to the Figure 4 to follow the evolution of the backscattered intensity with the refractive index. We can do the correlation with the evolution of the intensity vs. angles of backscattering for the same indices, illustrated by the Figure 5. We also can mention than the anisotropy coefficient g evolves from 0.80 for 1.35 to 0.77 for refractive index of the sphere equal to 1.85, measure of how the properties depend on the direction of examination. But, in spite of all these differences concerning the scattering properties, it is impossible to do the difference with the naked eye between the speckle patterns of these several media. It is well known that the naked eye like classical techniques carry out mainly the differentiation since being based on the intensities. That is the reason why informations may be hide in the phase. In order to demonstrate this point, we now do the comparison between three

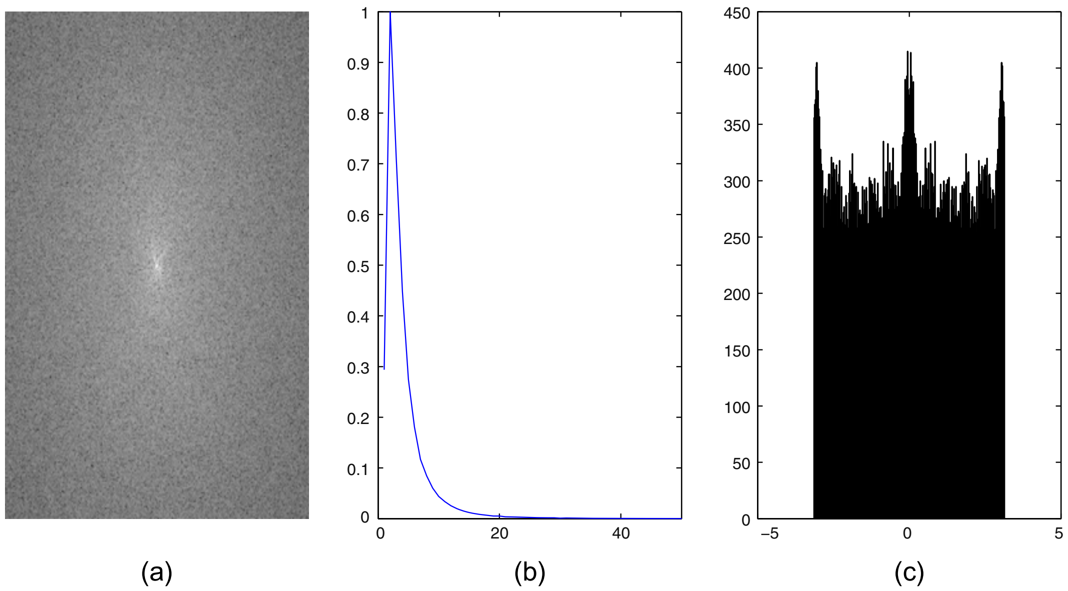

Fig. 2. Non normalized speckle pattern (a) and its statistics of intensity (b) and phase (c) respectively.

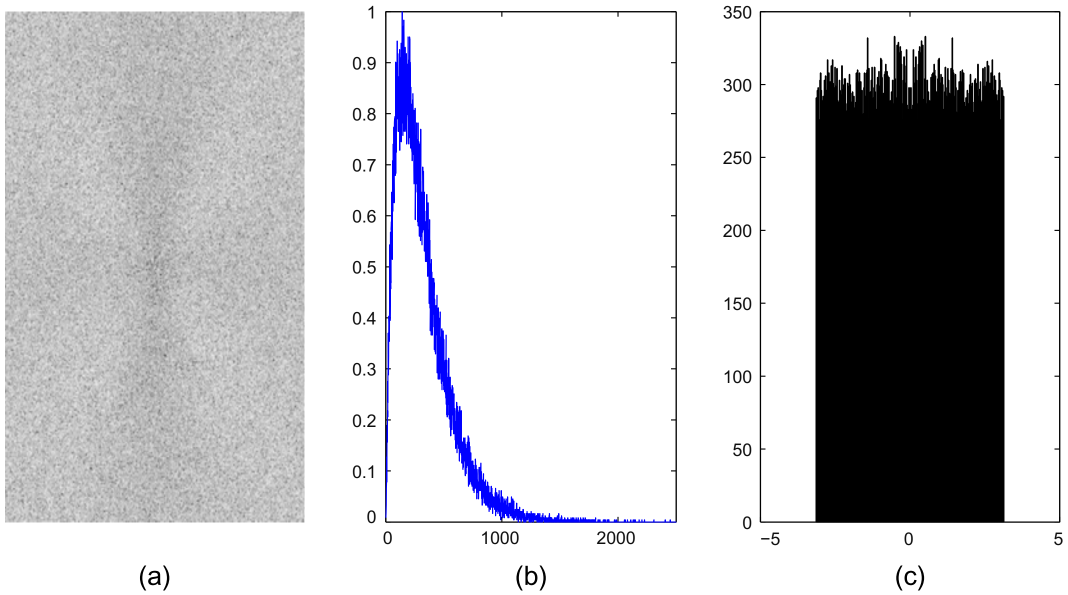

Fig. 3. Normalized speckle pattern (a) and its statistics of intensity (b) and phase (c) respectively (a.u.).

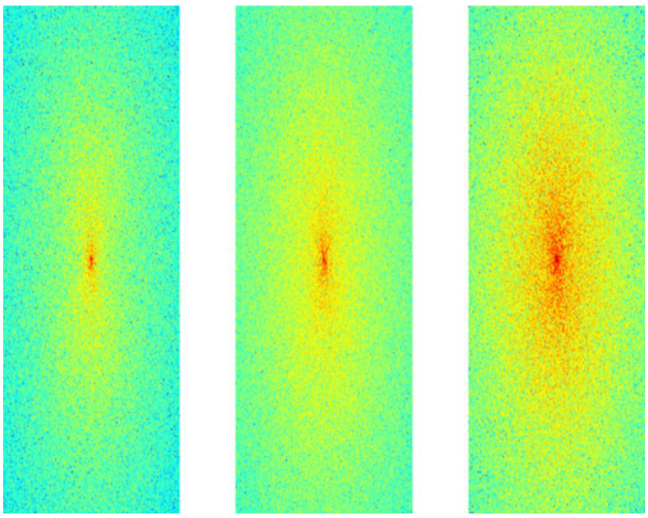

Fig. 4. Backscattered intensity (log scale) vs. refractive index (1.35, 1.55 and 1.75 respectively).

approaches. In a first time, we will try to discriminate these several media by the classical frequential approach. In a second time, we will apply our fractal based approach method [11] to this aim. Finally, we will present the results obtained with the phase spectrum increments approach.

4.1.1 Frequential based approach

We may refer to the Piederrière's paper [3] for a recall of this approach. It consists by the calculus of the power spectral density of the speckle following by the calculus of the autocorrelation function. This approach allow the measure of the average size of the speckle in the image. Unfortunately, this intensity based approach is unable, like our naked eye, to discriminate several speckle. In our case, the average size of the speckle stay constant and equal to eight pixels, so it only shows that our speckle is well developed.

4.1.2 Fractal based approach

This approach explicitly described in one of our previous paper [11] allows the characterization of any speckle pattern by three parameters. We called these one the Hurst coefficient, the saturation of variance and the size of the self similar element. Results obtained with this approach is summarized in the Table 1. Any tendencies may be extracted from these results; we can approximate that, while the variance increases linearly over the considered range, the other two parameters show oscillation around the mean value. Note that this sophisticated approach is also based on the intensity of the speckle pattern.

4.1.3 Phase based approach

To be concise, we just give through the Figure 6 is an example of the regression of the increments of the phase spectrum of a speckle pattern by the Von Mises distribution. On this Figure 6, we can see the increments of the phase spectrum in group of dots. The curve represents the regression of the group of dots by the Von Mises distribution. The error of the regression estimated by the maximum of variance is on average about 0.85%. According to the symmetry conditions of the simulation, we only calculate the spectrum of the phase in the ($\overrightarrow{x}$) direction of the speckle pattern (same results can be obtained in the ($\overrightarrow{y}$) one).

So, now, we gradually increase the refractive index of the sphere from 1.35 to 1.85. Each time, we note the both parameter of the Von Mises distribution i.e., μ and κ.

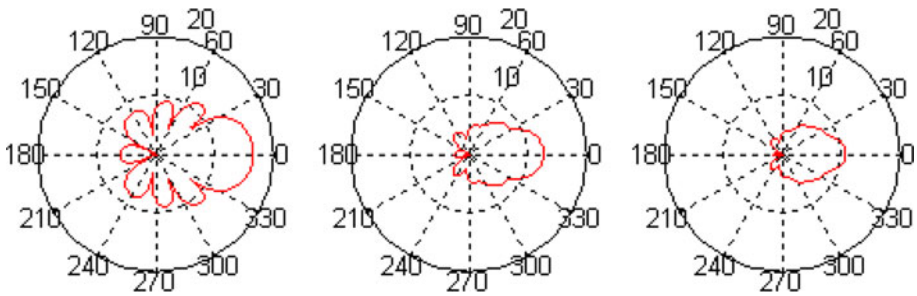

Fig. 5. Intensity (in log scale) vs. angles (refractive indices: 1.35, 1.55 and 1.75 respectively).

Table 1. Results of the fractal based approach.

Refractive indices	Variance	Hurst	Self-similarity
1.35	3.74	0.68	8.59
1.45	3.73	0.69	8.43
1.55	3.76	0.69	8.52
1.65	3.81	0.67	8.44
1.75	3.86	0.70	8.53
1.85	3.85	0.69	8.38

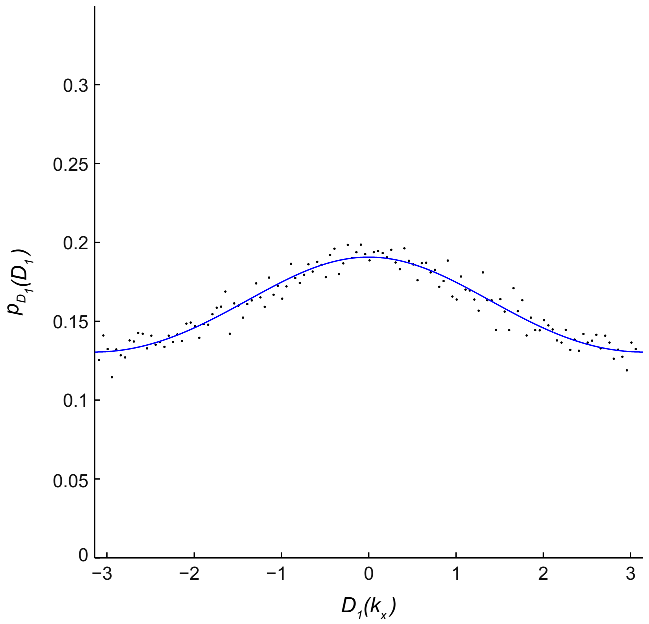

Fig. 6. Increments of the phase spectrum of a simulated medium and its regression by the Von Mises distribution.

Note that μ stay quasi constant and equals zero for all the presented media. Results on κ are presented in Figure 7.

The parameter κ decreases linearly with the refractive index. This simple observation is very significant. It makes possible to show that the increments of the spectrum of phase are narrowly correlated with the physical properties of the observed medium and thus allow, for the first time, to consider the resolution of the inverse problem, characterize a unknown samples considering the parameters of the associated Von Mises distribution. Moreover, this simple application demonstrates that phase

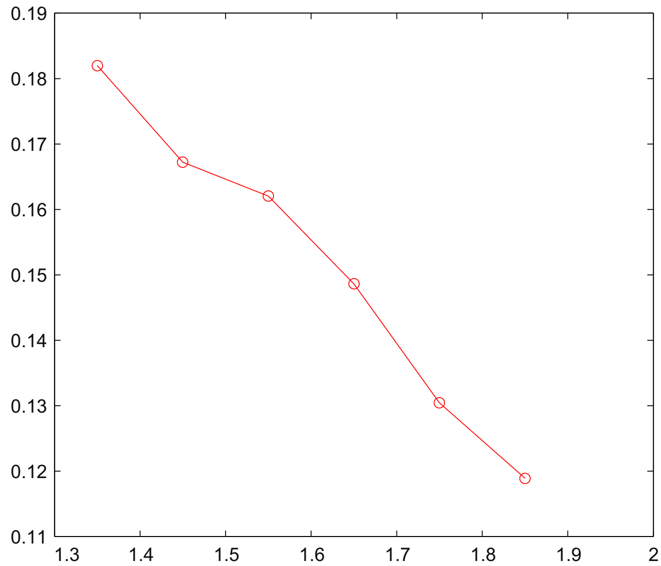

Fig. 7. Parameter κ of the Von Mises distribution vs. refractive indices.

based approach may be more useful than classical amplitude based approaches.

5 Conclusion

Even in the domain of image processing, the phase of the Fourier transformation is under-exploited even if it contains more informations than the amplitude spectrum. The reason seems to be its uniformity. So, in this paper, we propose for the first time, the exploitation of the phase spectrum of the speckle pattern for the characterization of scattering media. If the phase spectrum is uniform, it is not the same case for its increments which follow a so called "Von Mises distribution". So, the distribution of the increments of the phase spectrum can be characterized by indices which can be correlated with the physical properties of the media. This method, based on the phase, seems an interesting approach to complete another approach, based on the amplitude, that was presented previously [11]. Of course, we need now determine all the possibilities of this technique since first results on real test media are encouraging and will be the object of further paper. At all events, we hope that more consideration will be carried to the phase spectrum by the scientific community.

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