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Electromagnetic wave propagation in a randomly crystalline medium

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Abstract. Propagation of electromagnetic waves is considered in a crystalline solid medium with random parameters. We predict the generation of new exciton-type transversal and longitudinal waves caused by fluctuations of concentration and frequency.

PACS. 52.25.Gj Fluctuation phenomena – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion

1 Introduction

The effective dielectric permittivity tensor (EDPT) $\varepsilon_{ij}^{\text{eff}}(\omega, \mathbf{k})$ [1,2], associating Fourier transforms of the mean electric induction with the mean electric field strength, describes statistical properties of a randomly inhomogeneous medium. The advent of a length dimension parameter always leads to the excitation of new wave modes [1]. The EDPT for a crystalline medium consisting of an oscillators' assemblage with a random eigenfrequency and constant concentration has been obtained [3] using an effective medium approach [4–6]. It was shown that frequency fluctuations lead to the generation of the new exciton type transversal waves. Generalization of the problem for a weakly dispersive medium of oscillators with an eigenfrequency randomly fluctuating both in space and time has been considered in [7,8]. The EDPT of inhomogeneous heated plasma with random electron density fluctuations near the resonance frequency has been found by a quasi-hydrodynamic method [9]. An imaginary part of EDPT describes the scattering process of a regular component of the field into chaotic plasma and electromagnetic waves. The EDPT for a turbulent incompressible cold plasma stream at weak spatial-temporal electron concentration and velocity fluctuations has been obtained [10,11].

The investigation of the features of new exciton type longitudinal and transversal waves generated in a chaotically inhomogeneous crystalline solid medium consisting of oscillators assembled with a randomly varying space eigenfrequency and concentration is considered in this paper. Fluctuation of both parameters may be caused by polycrystallinity of the material or by some other aspect. Polycrystals and powders [11] are examples of such media.

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2 Formulation of the problem

The radiation of electromagnetic waves in random medium is governed by the stochastic Maxwell's equations and stochastically perturbed harmonic-oscillator equation

$$\ddot{\mathbf{S}} + \Omega^2(\mathbf{r})\mathbf{S} = \frac{e}{m}\mathbf{E}, \qquad \mathbf{j} = eN(\mathbf{r})\dot{\mathbf{S}}, \tag{1}$$

where $\mathbf{j}$, N and $\mathbf{S}$ are current density, concentration and displacement from equilibrium state, respectively. Eigenfrequency $\Omega(\mathbf{r})$ is a random function of position.

Splitting basic quantities into a coherent part plus an incoherent part

$$\begin{aligned}
\mathbf{E}(\mathbf{r},t) &= \langle \mathbf{E}(\mathbf{r},t)\rangle + \mathbf{E}_1(\mathbf{r},t),\\
\mathbf{S}(\mathbf{r},t) &= \langle \mathbf{S}(\mathbf{r},t)\rangle + \mathbf{S}_1(\mathbf{r},t),\\
\mathbf{N}(\mathbf{r}) &= N_0 + N_1(\mathbf{r}),\\
\Omega^2(\mathbf{r}) &\approx \omega_0^2 + \gamma_1(\mathbf{r}),\\
\gamma_1(\mathbf{r}) &= 2\omega_0\omega_1(\mathbf{r}),\\
N_0 &= const,\\
\omega_0 &= const,
\end{aligned} \tag{2}$$

where angular brackets denote an ensemble average. The mean value of the perturbation parts will be taken as zero. Substituting (2) into equation (1) and using the effective medium approach [4–6] we obtain the set of stochastic differential equations for the mean and random displacements [8]. All values are supposed to depend on time according to the law $\exp(\mathrm{i}\omega t)$ insofar as frequency and concentration fluctuations are functions of spatial coordinates. As a result the mean and the scattered fields satisfy

$$E_{ij}(\mathbf{r},\omega) = \frac{1}{(2\pi)^3}\frac{\omega^2}{c^2}\frac{\omega_\mathrm{p}^2}{\omega_0^2-\omega^2}E_j^0\exp(\mathrm{i}\mathbf{k}_0\cdot\mathbf{r}-\mathrm{i}\omega t)\int_{-\infty}^{\infty}\mathrm{d}\boldsymbol{\rho}\mathrm{d}\mathbf{k}\left\{\frac{\delta_{ij}}{\aleph^2-k^2}-\frac{1}{4\pi\aleph^2}\left[\left(\frac{4\pi}{3}\delta_{ij}-\frac{4\pi k_ik_j}{k^2-\aleph^2}\right)-\frac{4\pi}{3}\delta_{ij}\right]\right\}$$
$$\times\left[\frac{1}{\omega_0^2-\omega^2}\gamma_1(\mathbf{r}-\boldsymbol{\rho})-\frac{1}{N_0^2}N_1(\mathbf{r}-\boldsymbol{\rho})\right]\exp[\mathrm{i}(\mathbf{k}-\mathbf{k}_0)\cdot\boldsymbol{\rho}] \quad (5)$$

$$a(k_0,\omega) = \frac{\omega^4}{c^4}\left(\frac{\omega_\mathrm{p}^2}{\omega_0^2-\omega^2}\right)^2\frac{1}{\aleph^2}\left(\frac{1}{3}\tilde{W}(0,\omega)-\frac{\aleph^2}{k_0}\int_0^\infty\mathrm{d}\rho\left\{\sin k_0\rho\left[1-\frac{1}{\aleph^2\rho^2}(1-\mathrm{i}\aleph\rho)\right]\right.\right.$$
$$\left.\left.+\frac{1}{k_0\aleph^2}\frac{1}{\rho^3}\left(\frac{\sin k_0\rho}{k_0\rho}-\cos k_0\rho\right)(3-3\mathrm{i}\aleph\rho-\aleph^2\rho^2)\right\}\tilde{W}(\rho,\omega)\mathrm{e}^{\mathrm{i}\aleph\rho}\right), \quad (10)$$

$$b(k_0,\omega) = \frac{\omega^4}{c^4}\left(\frac{\omega_\mathrm{p}^2}{\omega_0^2-\omega^2}\right)^2\frac{1}{k_0^2\aleph^2}\int_0^\infty\mathrm{d}\rho\left[3\left(\frac{\sin k_0\rho}{k_0\rho}-\cos k_0\rho\right)-k_0\rho\sin k_0\rho\right]\frac{1}{\rho^3}(3-3\mathrm{i}\aleph\rho-\aleph^2\rho^2)\tilde{W}(\rho,\omega)\mathrm{e}^{\mathrm{i}\aleph\rho} \quad (11)$$

the equations [8]

$$\hat{L}\langle\mathbf{E}(\mathbf{r},\omega)\rangle = \frac{\omega^2}{c^2}\frac{\omega_\mathrm{p}^2}{\omega_0^2-\omega^2}\times\left[\frac{\langle\gamma_1(\mathbf{r})\mathbf{E}_1(\mathbf{r},\omega)\rangle}{\omega_0^2-\omega^2}-\frac{1}{N_0}\langle N_1(\mathbf{r})\mathbf{E}_1(\mathbf{r},\omega)\rangle\right] \quad (3)$$

$$\hat{L}\mathbf{E}_1(\mathbf{r},\omega) = \frac{\omega^2}{c^2}\frac{\omega_\mathrm{p}^2}{\omega_0^2-\omega^2}\times\left[\frac{\gamma_1(\mathbf{r})}{\omega_0^2-\omega^2}-\frac{1}{N_0}N_1(\mathbf{r})\right]\langle\mathbf{E}(\mathbf{r},\omega)\rangle, \quad (4)$$

where $\hat{L}\equiv\Delta-\mathrm{graddiv}+\aleph^2$,

$$\aleph^2 = \frac{\omega^2}{c^2}\left(1+\frac{\omega_\mathrm{p}^2}{\omega_0^2-\omega^2}\right) = \frac{\omega^2}{c^2}\frac{\Omega^2-\omega^2}{\omega_0^2-\omega^2} = \frac{\omega^2}{c^2}\varepsilon_0(\omega),$$

$\omega_\mathrm{p} = \left(4\pi N_0e^2/m\right)^{1/2}$ is the angular plasma frequency, $\Omega=(\omega_0^2+\omega_\mathrm{p}^2)^{1/2}$ is the hybrid frequency. Equation (4) can be solved with the aid of the appropriate Green's tensor of second rank. Submit the mean field as a monochromatic plane wave $\langle E_j(\mathbf{r}',\omega)\rangle = E_j^0\exp(\mathrm{i}\mathbf{k}_0\cdot\mathbf{r}'-\mathrm{i}\omega t)$. Hence the scattered field may be written in the form

see equation (5) above.

The average of the product of the random scalar functions at points separated in space are the autocorrelation functions

$$\begin{aligned}\langle\gamma_1(\mathbf{r})\gamma_1(\mathbf{r}-\boldsymbol{\rho})\rangle &= W_\gamma(\rho),\\ \langle N_1(\mathbf{r})N_1(\mathbf{r}-\boldsymbol{\rho})\rangle &= W_\mathrm{N}(\rho),\\ \langle\gamma_1(\mathbf{r})N_1(\mathbf{r}-\boldsymbol{\rho})\rangle = \langle N_1(\mathbf{r})\gamma_1(\mathbf{r}-\boldsymbol{\rho})\rangle &= W_{\gamma\mathrm{N}}(\rho).\end{aligned} \quad (6)$$

These correlation functions are taken as a function of the coordinate difference only, assuming that the statistical character of scalar parameters are homogeneous, but perhaps anisotropic. Here, homogeneity means that the dependence is upon coordinate difference only, and not on the values of the coordinates themselves; anisotropy means that there is a different functional dependence for each of the coordinate differences. Putting equation (5) in equation (3) and taking into account (6), we can obtain the equation for the mean field.

$$L_{ij}\langle E_j(\mathbf{r},\omega)\rangle = \frac{1}{4\pi}\frac{\omega^4}{c^4}\left(\frac{\omega_\mathrm{p}^2}{\omega_0^2-\omega^2}\right)^2E_j^0\exp(\mathrm{i}\mathbf{k}_0\cdot\mathbf{r}-\mathrm{i}\omega t)\times\left[\frac{4\pi}{3}\frac{1}{\aleph^2}\delta_{ij}\tilde{W}(0,\omega)-\int_{-\infty}^{\infty}\mathrm{d}\boldsymbol{\rho}\Lambda_{ij}(\boldsymbol{\rho})\tilde{W}(\rho,\omega)\right], \quad (7)$$

where

$$\begin{aligned}\tilde{W}(\rho,\omega) &= \frac{1}{N_0^2}W_\mathrm{N}(\rho)+\frac{1}{(\omega_0^2-\omega^2)^2}W_\gamma(\rho) - \frac{2}{N_0(\omega_0^2-\omega^2)}W_{\gamma\mathrm{N}}(\rho)\\ &= \left[\frac{\langle N_1^2\rangle}{N_0^2}+4\frac{\omega_0^2\langle\omega_1^2\rangle}{(\omega_0^2-\omega^2)^2}-4\frac{\omega_0\langle\omega_1N_1\rangle}{N_0(\omega_0^2-\omega^2)}\right]P(\rho)\\ &= Q(\omega)P(\rho),\end{aligned} \quad (8)$$

$P(\rho)$ is the regular function of ρ, $\langle\omega_1^2\rangle$ is the dispersion of frequency fluctuation, $\langle\omega_1N_1\rangle$ is the mutual correlation function of frequency and concentration fluctuations. From equation (7) it is straightforward to obtain the dispersion relation

$$(\aleph^2-k_0^2)\delta_{ij}+\mathbf{k}_i^0\cdot\mathbf{k}_j^0 = a(k_0,\omega)\delta_{ij}+b(k_0,\omega)\frac{\mathbf{k}_i^0\cdot\mathbf{k}_j^0}{k_0^2}, \quad (9)$$

where:

see equations (10, 11) above.

On the other hand from the equation for electric field it follows [2]

$$k_0^2\delta_{ij}-\mathbf{k}_i^0\cdot\mathbf{k}_j^0-\frac{\omega^2}{c^2}\varepsilon_{ij}^\mathrm{eff}(\mathbf{k}_0,\omega)=0, \quad (12)$$

$$\varepsilon_{ij}^{\mathrm{eff}}(\mathbf{k}_0,\omega) = \left(\delta_{ij} - \frac{\mathbf{k}_i^0 \cdot \mathbf{k}_j^0}{k_0^2}\right)\varepsilon_{\perp}^{\mathrm{eff}}(k_0,\omega) + \frac{\mathbf{k}_i^0 \cdot \mathbf{k}_j^0}{k_0^2}\varepsilon_{\parallel}^{\mathrm{eff}}(k_0,\omega).$$

Comparing equations (9, 12) and taking into account the explicit form of $\aleph^2$, we obtain

$$\varepsilon_{\parallel}^{\mathrm{eff}}(k_0,\omega) = \varepsilon_0(\omega) - \frac{c^2}{\omega^2}[a(k_0,\omega) + b(k_0,\omega)], \qquad (13)$$

$$\varepsilon_{\perp}^{\mathrm{eff}}(k_0,\omega) = \varepsilon_0(\omega) - \frac{c^2}{\omega^2}a(k_0,\omega). \qquad (14)$$

Hereafter the dispersion relations for longitudinal and transversal waves will be used [1,2].

3 Longitudinal waves

Let us consider small-scale inhomogeneities ($k_0\ell \ll 1$, ℓ is the characteristic spatial scale of inhomogeneities) of a crystalline random medium. Fast decrease of the correlation function allow us to trigonometrical expand functions into a series. This gives

$$\varepsilon_{\parallel}^{\mathrm{eff}}(k_0,\omega) = \left(\frac{c\aleph}{\omega}\right)^2 - \frac{1}{3}\left(\frac{\omega}{c\aleph}\right)^2\left(\frac{\omega_{\mathrm{p}}^2}{\omega_0^2-\omega^2}\right)^2 \times Q(\omega)\Big\{1 + \int_0^\infty \mathrm{d}\rho\Big(\frac{1}{5}k_0^2\aleph^2\rho^3 + \frac{2}{5}k_0^2\rho - 2\aleph^2\rho - \mathrm{i}\frac{2}{5}k_0^2\aleph\rho^2\Big)P(\rho)\mathrm{e}^{\mathrm{i}\aleph\rho}\Big\}. \qquad (15)$$

As in weakly dispersive medium $k_0\ell \ll 1$ and $\aleph\ell < 1$, imaginary part of equation (15) becomes

$$\mathrm{Im}\varepsilon_{\parallel}^{\mathrm{eff}}(k_0,\omega) \approx \frac{\sqrt{\pi}}{6}\varepsilon_0^{1/2}(\omega)\frac{\omega_{\mathrm{p}}^4}{(\omega_0^2-\omega^2)^2} \times \left(\frac{\omega\ell}{c}\right)^3 Q(\omega)\exp\left(-\frac{k_0^2\ell^2}{4}\right). \qquad (16)$$

In the no-frequency fluctuation condition, at $\omega_0 = 0$, $\omega \approx \Omega$, this expression transforms into a well-known result [9] obtained in the quasi-hydrodynamical approximation, where instead of the speed of light in a homogeneous medium there is the electron thermal velocity of a randomly inhomogeneous heated plasma. Pure imaginary component of the dielectric permittivity $\varepsilon_{\parallel}^{\mathrm{eff}}(k_0,\omega)$ describes energy transfer from the coherent component of the field to the stochastic oscillations of a randomly crystalline medium. Cubic dependence $\sim \ell^3$ of $\mathrm{Im}\varepsilon_{\parallel}^{\mathrm{eff}}(k_0,\omega)$ is characteristic [7–11] of a randomly inhomogeneous media with pure spatial fluctuations of parameters.

In a zero-order approximation in the absence of fluctuations from equation (13) it follows $\varepsilon_{\parallel}^{\mathrm{eff}}=(c\aleph/\omega)^2 = 0$ or $\omega^2 = \Omega^2$. In the presence of concentration and frequency fluctuations at $\omega = \Omega$, the wave number $\aleph$ vanishes. Therefore let $\aleph = 0$ in all terms including the exponent in the integrand of equation (15) and keeping the

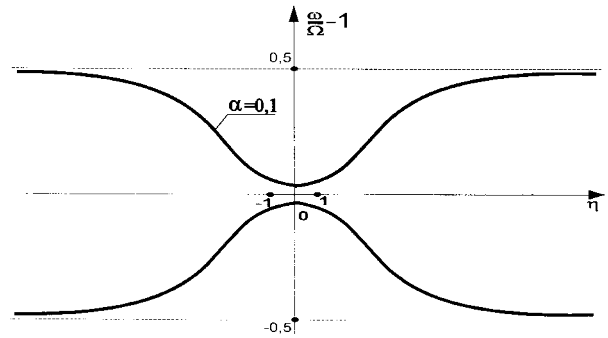

Fig. 1. Dispersion curves of longitudinal waves; $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.09$, $\varepsilon_3 = 0.08$.

terms $\sim \aleph^{-n}$ $(n > 0)$

$$\omega = \Omega + \delta \ (\delta \ll \Omega),$$
$$\omega^2 \approx \Omega^2 + 2\Omega\delta,$$
$$\Omega^2 - \omega^2 = -2\Omega\delta,$$
$$\varepsilon_0(\omega) = \frac{2\delta\Omega}{\omega_{\mathrm{p}}^2},$$
$$\aleph^2 = \frac{2\delta\Omega^3}{c^2\omega_{\mathrm{p}}^2}.$$

Hereafter we will limit ourselves by consideration of an isotropic case and suppose that all correlation functions have a Gaussian form $P(\rho) = -(\rho^2/\ell^2)$ with the same characteristic spatial scale ℓ (in general case three correlation functions may be described by various functions and characteristic spatial scales). Using the dispersion relation for longitudinal waves we obtain

$$\omega = \Omega \pm \frac{1}{2\sqrt{3}}\frac{\omega_{\mathrm{p}}^2}{\Omega}\Big(\frac{\langle N_1^2\rangle}{N_0^2} + 4\frac{\omega_0^2\langle\omega_1^2\rangle}{\omega_{\mathrm{p}}^4} + 4\frac{\omega_0\langle\omega_1 N_1\rangle}{N_0\omega_{\mathrm{p}}^2}\Big)^{1/2} \times \left(1 + \frac{1}{10}k_0^2\ell^2\right). \qquad (17)$$

The signs ($\pm$) correspond to the high- and low-speed modes. If concentration fluctuations are absent, equation (17) goes over into the equation which has been obtained in [3]. Figure 1 shows dispersion curves – normalized frequency-nondimensional parameter $\eta = k_0\ell$ – relation, in particular, the mutual correlation function is dominant. The group velocity of an exciton-type longitudinal wave linearly depends on k_0 and its direction is determined also by the sign of $\langle\omega_1 N_1\rangle$. So, the mutual correlation function of two random scalars (concentration and frequency) may have a substantial effect as on attenuation (or growth) of the longitudinal waves as well as on the change of direction of the group velocity relative to the wave vector $\mathbf{k}_0$.

Using equations (10, 11, 13) in the case of large-scale inhomogeneities, $k_0\ell \gg 1$ we get

$$\omega = \Omega \pm \frac{\omega_{\mathrm{p}}^2}{2\Omega}\left[1 + \frac{1}{3}Q(\omega)\right]^{1/2}.$$

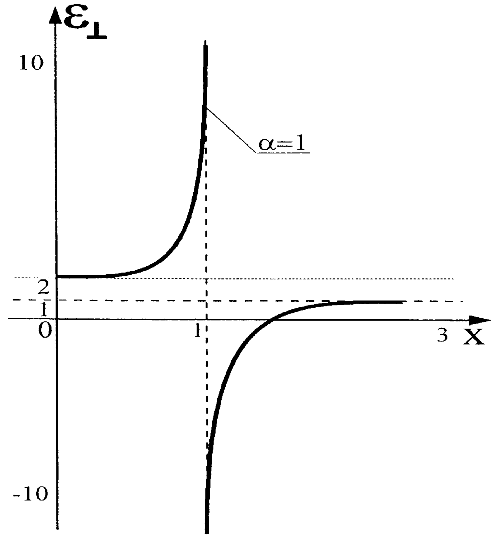

Fig. 2. Illustrating the behaviour of transversal permittivity $\varepsilon_\perp(X) = 1 + (\alpha^2 - X^2)^{-1}$ as a function of $X = \omega/\omega_\text{p}$; $\alpha = \omega_0/\omega_\text{p} = 1$.

4 Transversal waves

The dispersion relations for transversal waves may be obtained from equations (14, 10) in the frequency band near the resonance frequency $\omega \approx \omega_0$ at $Q(\omega) \ll 1$:

$$\varepsilon_\perp^{(1)\text{eff}}(\omega) = 1 + \frac{\omega_\text{p}^2}{\omega_0^2 - \omega^2}, \tag{18}$$

$$\varepsilon_\perp^{(2)\text{eff}}(\omega) = 1 - \omega_\text{p}^2(\omega_0^2 - \omega^2)\Big[4\omega_0^2\langle\omega_1^2\rangle - 4\frac{\omega_0\langle\omega_1 N_1\rangle}{N_0}(\omega_0^2 - \omega^2) + \frac{\langle N_1^2\rangle}{N_0^2}(\omega_0^2 - \omega^2)^2\Big]^{-1}. \tag{19}$$

The dispersion relation (18) describes transversal waves in an optical isotropic media [1]. In particular, near the dipole line, in the case of an ideal gas, the frequency ω_0 must be replaced by the jump frequency $\omega_{0\text{s}} = \omega_\text{s} - \omega_0$ from the s state into the equilibrium one; angular plasma frequency ω_p must be replaced by $\Omega_0 = \left((4\pi Ne^2/m)f_{0\text{s}}\right)^{1/2}$, $f_{0\text{s}}$ is the oscillator strength, e and m are charge and mass of an electron. A plot of $\varepsilon_\perp^{(1)\text{eff}}(X)$ as a function of nondimensional parameter $X = \omega/\omega_\text{p}$, at $\alpha = \omega_0/\omega_\text{p} = 1$ is illustrated in Figure 2. Numerical simulations show that decreasing the parameter by an order of magnitude ($\alpha = 0.1$) the bandwidth increases by the second degree of order. The dispersion relation (19) transforms into the well-known one [3] in the absence of concentration fluctuations. Let us rewrite equation (19) through the nondimensional parameters: $\varepsilon_1 = \langle N_1^2\rangle/N_0^2$, $\varepsilon_2 = \langle\omega_1 N_1\rangle/\omega_\text{p}N_0$, $\varepsilon_3 = \langle\omega_1^2\rangle/\omega_\text{p}^2$. Figure 3 illustrates dispersion characteristics of these waves in case of chaotically inhomogeneous crystalline medium in the absence of concentration fluctuation ($\varepsilon_1 = \varepsilon_2 = 0$, $\varepsilon_3 = 0.1$). Numerical calculations were carried out at various values of the parameter α. The curves have a parabolic form and their branches cross the axis of abscissae. Truncational frequencies have appeared and the crystalline medium behaves as a filter. Numerical simulations show that growth of the oscillator's eigenfrequency, *i.e.* increase of the parameter α, leads to the displacement of the truncational frequency to the high-frequency range. Dielectric permittivity $\varepsilon_\perp^\text{eff}$ *versus* X at $\alpha = 1$ and in the absence of frequency fluctuations ($\varepsilon_1 = 0.03$, $\varepsilon_2 = \varepsilon_3 = 0$) is given in Figure 4. Resonance regions have appeared at frequencies $\omega = \pm\,\omega_\text{p}$. The frequency dependence of transversal dielectric permittivity is illustrated graphically in Figure 5 for the case in which $\omega_0 = 7\omega_\text{p}$ ($\alpha = 7$), $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.09$, $\varepsilon_3 = 0.08$. New resonance regions and truncational frequencies appear in this case.

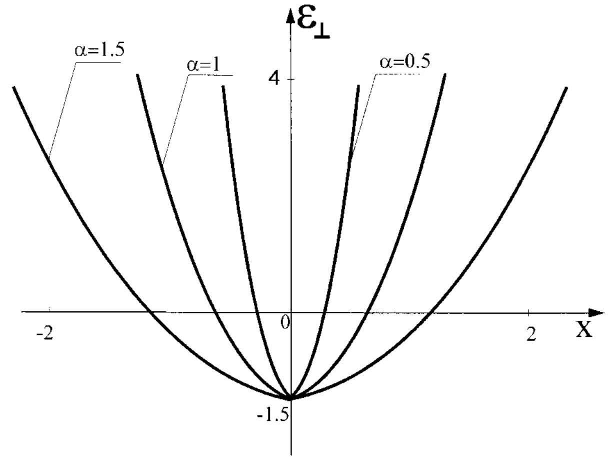

Fig. 3. Plots of $\varepsilon_\perp(X) = 1 - [(\alpha^2 - X^2)/4\alpha^2\varepsilon_3]$ *versus* X without concentration fluctuations; $\varepsilon_3 = \langle\omega_1^2\rangle/\omega_\text{p}^2 = 0.1$.

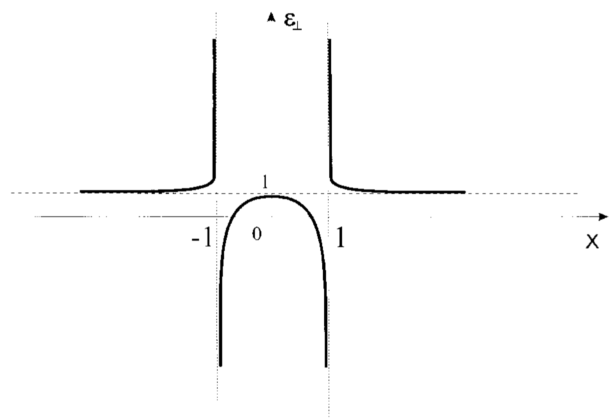

Fig. 4. Plots of $\varepsilon_\perp(X) = 1 - \varepsilon_1^{-1}(\alpha^2 - X^2)^{-1}$ *versus* X without frequency fluctuations; $\varepsilon_1 = \langle N_1^2\rangle/N_0^2 = 0.03, \alpha = 1$.

5 Conclusion

The scalar harmonic oscillator equation with a stochastic frequency is the starting point of this paper. We have developed a theory for the propagation of electromagnetic waves in media with frequency and concentration fluctuations. The statistical properties of an electromagnetic

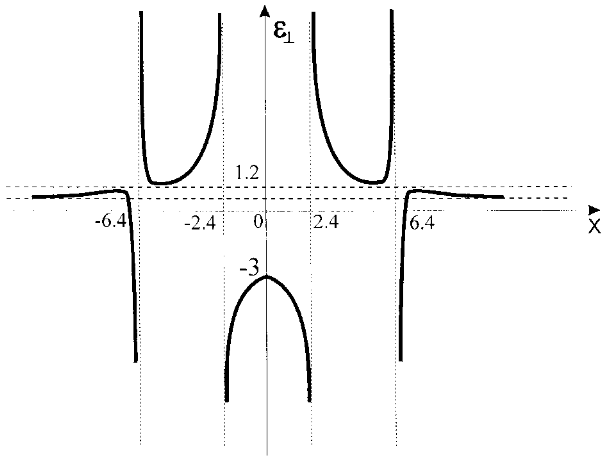

Fig. 5. Plots of $\varepsilon_\perp(X) = 1 - (\alpha^2 - X^2)[4\alpha^2\,\varepsilon_3 - 4\alpha\,\varepsilon_2(\alpha^2 - X^2) + \varepsilon_1(\alpha^2 - X^2)^2]^{-1}$ *versus* X. The curves are calculated for the case in which $\varepsilon_1 = 0.05$, $\varepsilon_2 = \langle\omega_1 N_1\rangle/\omega_\mathrm{p} N_0 = 0.09$, $\varepsilon_3 = 0.08$, $\alpha = 7$.

wave propagating through a randomly crystalline medium are analyzed by an effective medium approach. The effective dielectric tensor can be explicitly considered for a arbitrary spatial correlation function. Dispersion relations are analyzed analytically and numerically for the case of a Gaussian spatial correlation function. It is shown that frequency and concentration fluctuations lead to the generation of new excitation type transversal electromagnetic waves. Their parameters are expressed through the statistical characteristics of fluctuating parameters. Mutual correlation of these random scalar functions lead to the appearance of new resonance regions and truncational frequencies. It is anticipated that in an account of the temporal pulsations of a medium's parameters, with replacement of correlation functions, characteristic spatial and temporal scales of inhomogeneities will play an important role in the generation of new exciton-type waves in randomly crystalline media.

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