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Thin wire approximation for PCB modeling of the EMC of power converters

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Abstract. This paper deals with the modeling of Printed Circuit Boards (PCB) for numerical resolution of a power circuits electromagnetic Compatibility (EMC). The well-adapted model for numerical resolution is the thin wire model. Thus, the PCB trace is replaced by N thin wires. Each wire radius is calculated as a function of the PCB dimensions. This model is applied to calculate the radiated electric field from a PCB circuit using the moment method which is well-suited for thin wire modeling. By observing the influence of the electromagnetic field radiated from the circuit, the number of equivalent wires required to have coherent results is established. The study of the current distribution in the wires shows that the thin wire model takes into account both the skin and the proximity effects.

PACS. 84.30.Bv Circuit theory (including computer-aided circuit design and analysis) – 84.40.Ba Antennas: theory, components and accessories – 85.40.Bh Computer-aided design of microcircuits; layout and modeling

1 Introduction

The use of circuits with very fast converters makes the radiated emission control, according to international EMC regulation, a very difficult task for power electronic circuit designers. Generally, electric apparatus is realized on Printed Circuit Boards (PCB). PCB can be in simple or double faces. They are used to fix electric components, to drive current to components and to transmit signals. Although PCB layout is as important to the reduction of EMC problems as wiring layout it is often neglected. The study of the PCB layout is today necessary to analyze conducted/radiated emissions and immunity. PCB traces have rectangular sections with disproportion in dimensions (the ratio between width and thickness of the rectangular is about 100 and the ratio between width and length of the PCB is greater than 1000). Consequently, the modeling of this kind of conductor is very difficult and usually impossible using numerical methods. Thereby, the use of an equivalent model for PCB traces is recommended.

This work is a part of an ongoing research project with the ultimate aim to develop a prediction tool for radiated EMI from circuits of static converters. This tool will be based on a computational method, enabling emission level prediction during the design phase in order to verify the compliance of the future circuit with international standards regulations.

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There are several computational methods in electromagnetic compatibility. The most famous are: Finite element method, Finite difference time domain method, Transmission line matrix method and the moment method. The choice of the numerical method depends on several parameters and not only those considered in this study. A previous assessment [1] shows that in the area of electromagnetic compatibility in power electronic circuits, the moment method has an advantage over all others: it gives a direct and accurate solution of the current distribution. Furthermore, this method does an excellent job of analyzing unbounded radiation problems and it excels in analyzing perfect electric conductor (PEC) configurations. It considers two-dimensional and three-dimensional conducting structures that can be represented by wires having different shapes and radiuses. The effects of ground can be investigated in two cases: perfectly or imperfectly conducting ground. Both lumped and distributed loading can be taken into account and the wire can be excited at any arbitrary point along its length. In the case of wire modeling, the integral equation solved by the moment method is that of the electric field.

2 Printed circuit board modeling

The most famous model for PCB traces is the thin wire model. The thin wire approximation consists of replacing the rectangular conductor, which has dimensions w and e, by N cylindrical conductors. A cylindrical conductor is characterized by its radius a (Fig. 1).

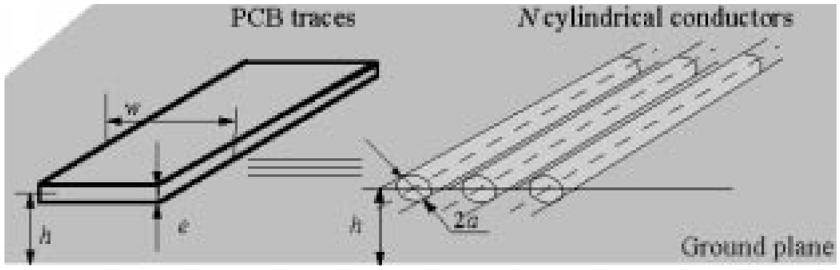

Fig. 1. The thin wire approximation.

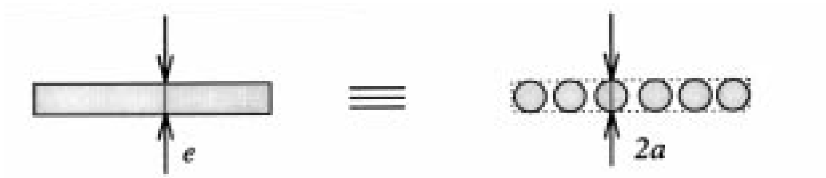

Fig. 2. The wire diameter is equal to the thickness e ($2a = e$).

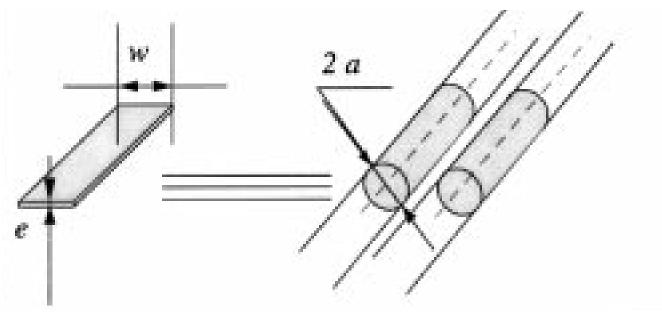

Fig. 3. The wire diameter is calculated using the PCB dimensions and N.

This approximation is valid only if the radius of the wire, the distance between wires and the height h separating the conductors from the ground plane are small compared to the wavelength. The number of wires and the radius of each one must be chosen in a way that the radiation from the wires is equivalent to that from the PCB. To satisfy this latter requirement, we will first recall the approach adopted by Petit [2] and then propose another approach.

2.1 Wire diameter is assumed to be equal to the thickness e

In this model, the conductor thickness is conserved. So the PCB is replaced by N wires with diameters which are equal to the PCB thickness e (Fig. 2). This model had been studied by Petit [2]. Using transmission line theory, Petit determined the number N of conductors necessary to satisfy the condition of conserving current and voltage at the entrance and the exit of the conductor. He demonstrated that at least four wires must be used to have the same voltage and current as those of a PCB trace which is one-millimeter in width. He also showed that the results are as much accurate as the number of conductors, and the distance to the ground plane is important. However, he noted that the wires must be equidistant to take into account the skin effect.

Nevertheless, the number of conductors required is important and we propose to adopt another assumption to reduce this number. In fact, the number of conductors must be optimized to reduce the computation time and the number of unknowns.

2.2 Wire diameter is calculated using the PCB dimensions and N

The PCB trace (width w, thickness e) will be replaced by N cylindrical conductors having a diameter $2a$ (Fig. 3). The diameter $2a$ depends on w, e and N. When transforming the rectangular section into a cylindrical one, we propose two different conditions to be satisfied:

- first, the conductor resistance should be preserved. Hence, the section area of the equivalent model should be the same as that of the PCB trace. So, $we = N\pi a^2$;
- second, since the frequency is large, a skin effect may take place, therefore, the current is expected to be superficial and the conductor's sectional circumference should be conserved. Hence, $w + e = N\pi a$.

To satisfy these conditions, we will calculate the radius as an average of both the equivalent radiuses proposed above. Consequently, for an equivalent model constituted by N wires, the diameter $2a_N$ is obtained using the following formula:

$$2a_N = \frac{w+e}{\pi N} + \sqrt{\frac{we}{\pi N}}\,. \qquad (1)$$

In the following, we will apply the thin wire approximation for modeling PCB traces with moment method.

3 Moment method for thin wire modeling

The method of moments has been in use for many years for a wide variety of applications. For example, it is very commonly used to analyze antenna structures, for which it continues to be the most popular, especially when the topology can be represented by thin wire geometry and surfaces [3].

3.1 Current distribution and radiated field

Consider a surface S of a perfect conductor, in free space, as shown in Figure 4. The boundary condition at the surface of the conductor is:

$$\mathbf{n} \times \mathbf{E}_T = \mathbf{0} \tag{2}$$

where $\mathbf{n}$ is the vector normal to the surface of the conductor and $\mathbf{E}_T$ is the total electric field consisting of both incident field $\mathbf{E}_i$ and scattered field $\mathbf{E}_s$:

$$\mathbf{E}_T = \mathbf{E}_s + \mathbf{E}_i. \tag{3}$$

The condition expressed by equations (2, 3) can be written as:

$$\mathbf{n} \times \mathbf{E}_s = -\mathbf{n} \times \mathbf{E}_i. \tag{4}$$

The scattered field $\mathbf{E}_s$ is defined as the field produced by all currents and charges present in the conductor:

$$\mathbf{E}_s = -j\omega\mathbf{A} - \boldsymbol{\nabla}\boldsymbol{\Phi} \tag{5}$$

where $\mathbf{A}$ is the magnetic field vector potential:

$$\mathbf{A} = \frac{\mu}{4\pi}\int_S \mathbf{J}(\mathbf{r}_1)\frac{e^{-jkR}}{R}ds \tag{6}$$

and $\boldsymbol{\Phi}$ is the electric field scalar potential:

$$\boldsymbol{\Phi} = \frac{1}{4\pi\varepsilon}\int_S \rho(\mathbf{r}_1)\frac{e^{-jkR}}{R}ds. \tag{7}$$

$\mathbf{J}(\mathbf{r}_1)$ is the current distribution on the conductor, R is the distance from the current source point to that where the field is to be evaluated, S is the surface of the conductor, $k = 2/\pi\lambda$ and ρ is the charge density:

$$\rho(\mathbf{r}_1) = -\frac{1}{j\omega}\mathrm{div}_S(\mathbf{J}(\mathbf{r}_1)) \cdot \tag{8}$$

In thin wire approximation (Fig. 5), it is assumed that:

- the wire radius a is small compared to the wavelength but the ratio of a to the wire length L need not be small. Hence, the current flows only in the axial direction for each wire. This means that the circumferentially directed current is small;
- current and charge densities are approximated by a filamentary current I and charge ρ along the wire axis.

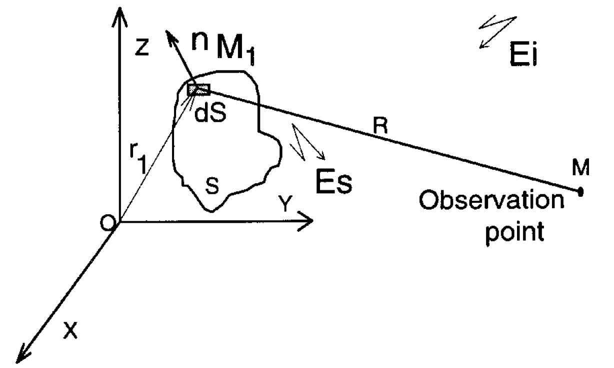

Fig. 4. Boundary value problem. General conductor.

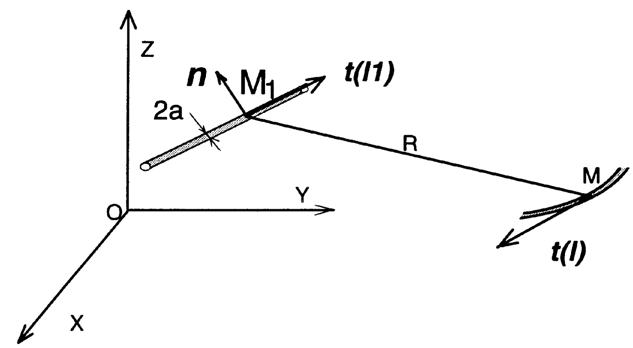

Fig. 5. Boundary value problem. Wire approximation.

The current distribution will then be modeled as an infinitely thin sheet of current forming a tube of radius a, with the density of current independent of circumferential position on the tube.

Using equations (4–8) we can write:

$$t(l)E_i = j\omega\mu\int_{wire} t(l)t(l_1)I(l_1)g(R)dl_1 - \frac{1}{j\omega\varepsilon}\frac{d}{dl}\int_{wire}\frac{dI(l_1)}{dl_1}g(R)dl_1 = L[I(l)] \tag{9}$$

where g is the free space average of the Green's function over the wire circumference:

$$g(R) = \frac{1}{2\pi a}\int_C \frac{e^{-jkR}}{4\pi R} \tag{10}$$

$\mathbf{t}(l)$ is the tangential vector, l is the length variable along the axis of the conductor (Fig. 5) and C is the circumference of the conductor.

The entire structure of the problem can be broken down into wires or metal plates. Thus, equation (9) is the electric field integral equation for current distribution along these wires. To solve this equation we use the moment method technique.

3.2 Moment method technique

Each wire of the structure is subdivided into a number of wire segments, which must be small compared to the wavelength. Once the model is defined, a source is imposed (a plane wave approaching or a voltage source on one of the wire segments). The moment's technique is used to determine the current on every wire segment due to the source and all the other currents. Once these currents are known, then the electric field at any point in space is determined from the sum of the contribution of each wire segment.

The unknown function is the current $I(l)$ over the structure, it can be written as:

$$I(l) = \sum_{j=1}^{N} I_j F_j(l), \tag{11}$$

where $F_j(l)$ are the current basis functions.

So, equation (9) can be written in a matrix form:

$$[Z_{ij}][I_j] = [V_i], \tag{12}$$

where:

$Z_{ij} = \langle \omega_i, L(F_j) \rangle$ is the impedance matrix,

$V_i = \langle \mathbf{E}_i, \mathbf{t}(l) \rangle$ is the excitation voltage vector,

and $\omega_1, \omega_2, \omega_3$,... are N weighting functions.

Basis and weighting functions are often sinusoidal or triangular functions. In the present investigation, the basis functions are sinusoidal whereas the weighting functions are delta functions. To model the ground plane, we suppose that it has a perfect conductivity and infinite dimensions, and thus we will use the image theory. Finally, to simplify the problem we will neglect the effect of the dielectric supporting the conductors.

4 Application

4.1 Description of simulated structures

In this study, we use the moment method to model a simple circuit shown in Figure 6. It is constituted by a PCB trace, which is 100 mm in length, 5 mm in width, and 35 μm thick. It is at a distance h from the ground plane.

The conductor is excited at one of its extremities by a voltage source of 1 V at a frequency f. The other extremity is connected to the ground plane.

4.2 Equivalent model

The equivalent model is shown in Figure 7. It is constituted by N thin wires. The wire diameter is calculated as explained above using equation (1). In Table 1, we give the calculated equivalent radius for different values of N.

The number of equivalent conductors N must be optimized to minimize the number of unknowns and calculation time. For this reason, we will study the influence of the number N on the precision of the results.

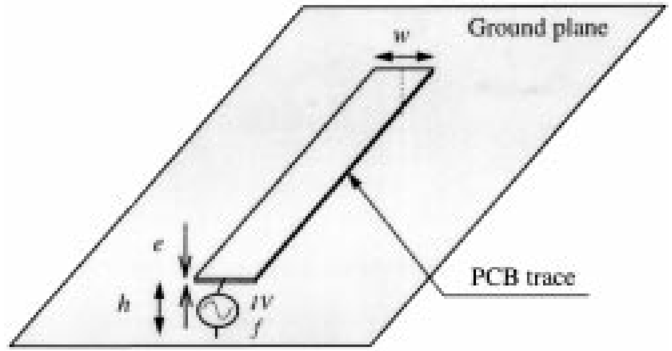

Fig. 6. Circuit to be modeled.

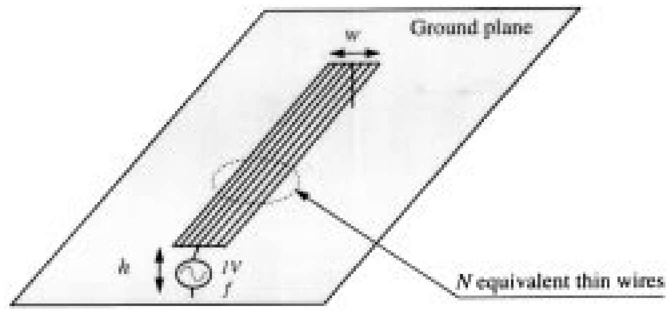

Fig. 7. The equivalent model.

4.3 Influence of the number N of wires on radiated electromagnetic field

We consider the point situated at one meter above the center of the PCB trace. In Figure 8, we present the radiated electric field at this point. The PCB trace is at 1.5 mm from the ground plane. Results are obtained using the moment method [4] for three values of the frequency 1 MHz, 10 MHz and 100 MHz. For each frequency we calculate the radiated electric field with a number of wires going from 1 wire to 21 wires. It appears that the radiated electric field increases considerably with N if N is less than 5, then it varies slowly with N to reach an asymptotic value obtained once the conductors' number is equal to 7. Thus, values of calculated electric field with 11 wires are almost the same as those obtained by 21 wires. So, we conclude that we must have at least 7 wires to obtain convergent results when modeling a PCB with width equal to 5 mm.

On the other hand, we remark that if the excitation frequency increases the convergence value of the electric field increases. This behavior is physically correct. We notice also that if the frequency increases, the convergence of results is obtained for fewer wires. This can be explained by the fact that when calculating electric field by moment method numerical errors are more important when the frequency is lower. This phenomenon is more observable with the magnetic field in Figure 10.

When comparing the curves obtained with different values of the height h from the ground plane (Fig. 8: $h =$ 1.5 mm and Fig. 9: $h = 15$ mm) we note that if the height

Table 1. Calculated equivalent radius for different values of N.

Wires' Number N	1	2	3	5	7	11	21
Equivalent radius a_N (mm)	0.92	0.49	0.34	0.21	0.16	0.11	0.065

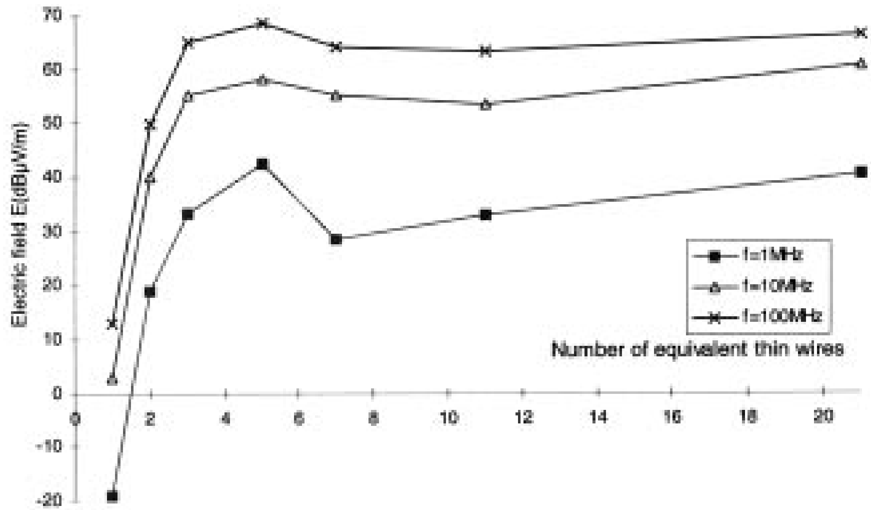

Fig. 8. Influence of wires' number on the radiated electric field, $h = 1.5$ mm.

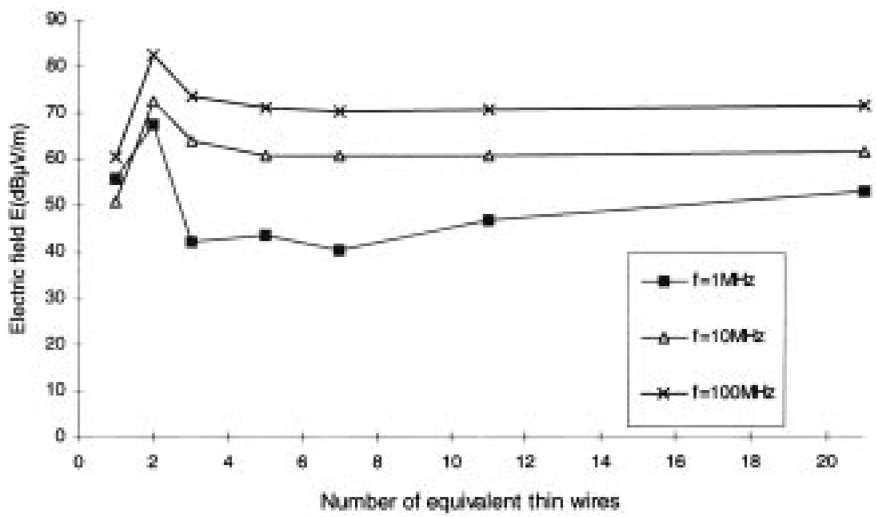

Fig. 9. Influence of wires' number on the radiated electric field, $h = 15$ mm.

h increases, the values of radiated electric field increase also and their convergence becomes better.

This latter phenomenon can be explained as follows: if h increases, coupling between the wires and ground plane decrease, so errors caused by thin wire approximation decrease, and finally the number of thin wires required to realize the convergence is smaller.

The same results about the convergence of the radiated electric field are obtained for the radiated magnetic field and showed in Figure 10. Results are obtained for a height $h = 15$ mm and for the same values of frequency as those used above.

4.4 Current Distribution, edge effect and skin effect in PCB traces

In this part of the paper, we are interested in the current distribution across the width of the PCB. Hence, we will study the current distribution in a section of the wires. In Figure 11, we present the amplitude of the current calculated at points situated on different wires of the same cross-section, for a frequency generator of 100 MHz. The PCB, with 5 mm width, is modeled by seven equidistant wires.

Current distribution shows that the model of thin wire takes into account the edge effect between all the wires. This edge effect is due to an inductive crosstalk. The edge

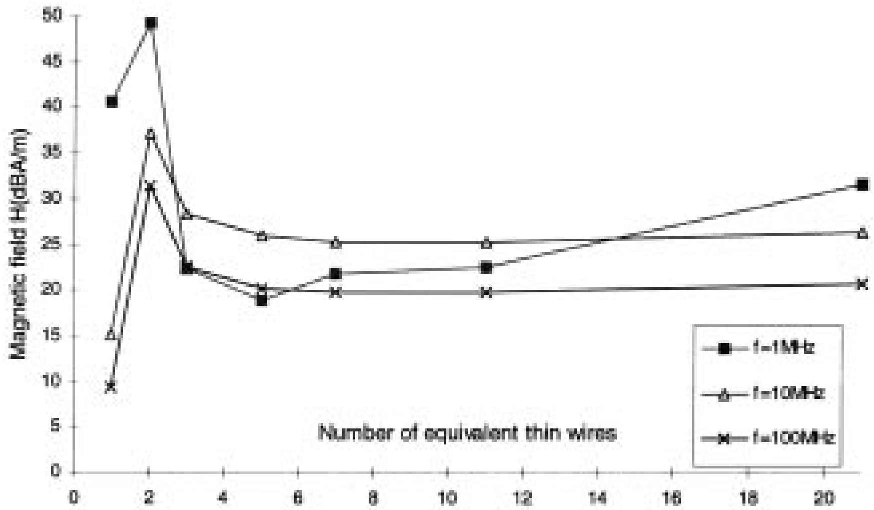

Fig. 10. Influence of wires' number on the radiated magnetic field, $h = 15$ mm.

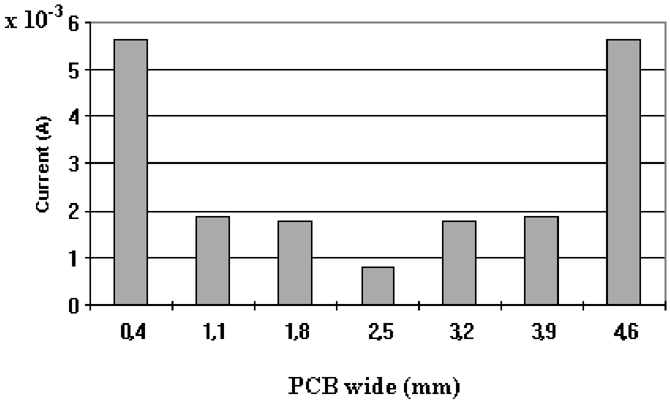

Fig. 11. Current distribution inside thin wires $f = 100$ MHz, $h = 15$ mm.

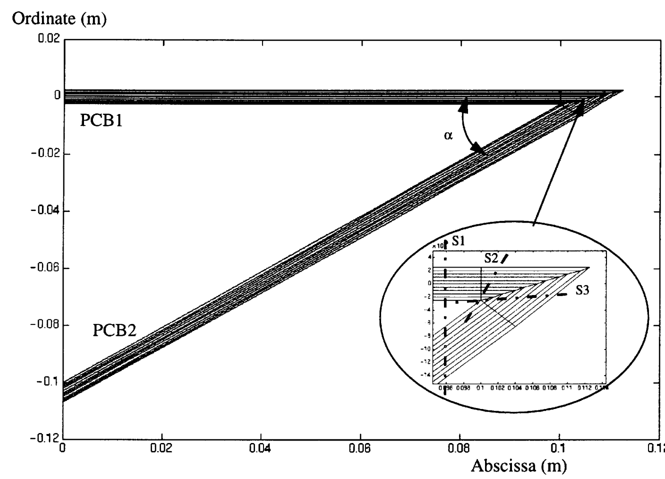

Fig. 12. Circuit with two PCB traces.

effect between wires of the model represents the skin effect which is due to the eddy currents in the PCB trace section. In fact, if the excitation frequency increases, the current in wires situated at extremities increases and becomes more important than that of wires placed at the middle of the section.

4.5 Proximity effect in PCB traces

This part of the study focuses on the proximity effect in PCB traces. For this task, we model a circuit constituted by two PCB traces situated at the same distance from the ground plane. The two conductors are linked and make between themselves an angle α. An extremity of the first PCB is connected to a voltage source of 1 V at a frequency $f = 10$ MHz whereas the second PCB trace is connected to the ground plane. Each PCB trace is replaced by 11 thin wires (Fig. 12). The wires' radiuses are computed as explained above.

In Figures 13, 14 and 15, we present the current distribution in conductors across respectively sections S1, S2 and S3.

When we compare the current distribution in PCB1 and PCB2 to that obtained for a simple PCB trace, we note some changes in the distribution. Thus, the latter is not only defined by the skin effect but also by a proximity effect due to coupling between the two PCB traces. Consequently, as shown in Figure 13 the main part of the current in the PCB1 is conducted in the wire located on the same side of the PCB2. This phenomena can be explained by the fact that current in PCB1 and PCB2 are in the opposite direction. Thereby, coupling between the two PCB traces reduces the total impedance of the wire described previously. On the other side, Figures 14 and 15 show a prominent current in the elbow. An inductive crosstalk appears between all the loops in the elbow (Fig. 16). These little loops have small impedance; therefore the induced current (i_1) in each loop is important which is why we obtain negative current in some wires of the elbow.

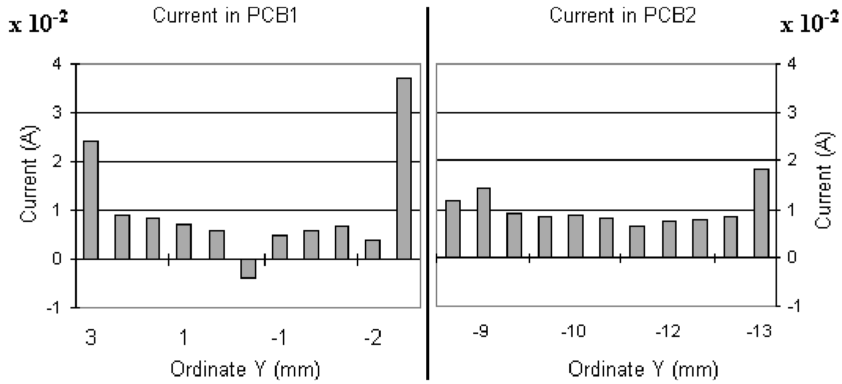

Fig. 13. Current distribution in wires of the section S1.

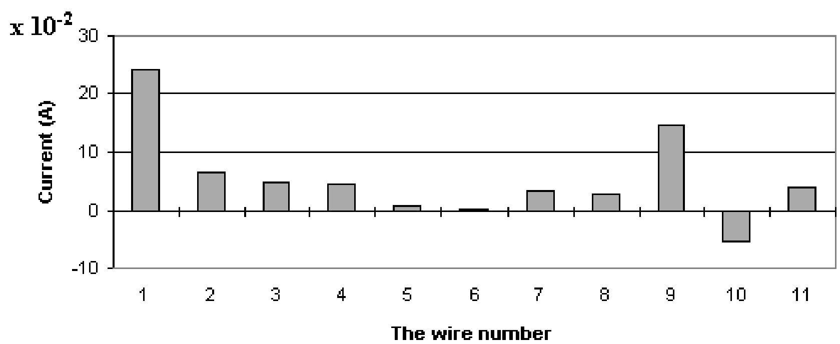

Fig. 14. Current distribution in wires of the first part of the elbow (section S2).

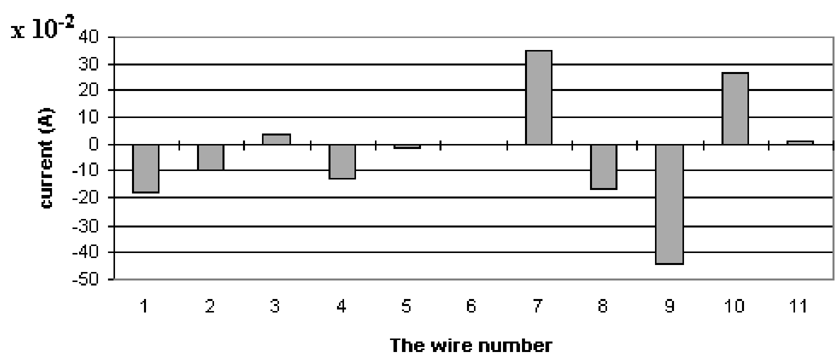

Fig. 15. Current distribution in wires of the second part of the elbow (section S3).

5 Conclusion

In PCB modeling, the use of thin wires is essential to avoid numerical method limitations. On the other hand, the number of thin wires of the model must be optimized to minimize the calculation time and the number of unknowns to solve.

The study of the influence of the number of conductors on the electric and magnetic field radiated from a simple circuit shows that for a PCB trace with a width equal to five millimeters, the number of wires must be greater than 7 to have convergent results.

The study of the current distribution along wires situated at the same section of a PCB trace shows that the thin wire model takes into account the skin effect in conductors. On the other hand, the study of a circuit constituted by two PCB traces displays the proximity effect in conductors.

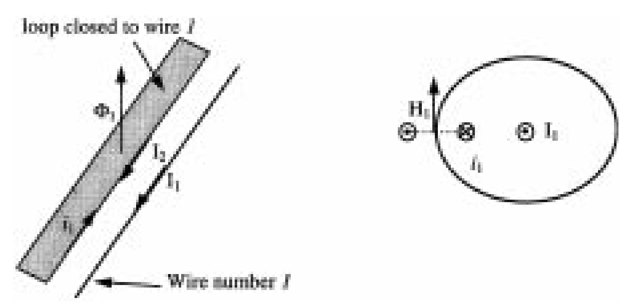

Fig. 16. Inductive crosstalk between wires.

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