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# Pressure self-multiplication and the kinetics of phase transition in plastic layer experiencing plane deformation

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**Abstract.** Based on the deformation theory of plasticity the problem of pressure distribution in a compressed layer at phase transition experiencing a plane plastic deformation is considered. It is found that in the pressure distribution near the phase boundaries anomalies emerge in the form of a "step" or a local maximum caused by volume jumps at phase transition. It is shown that these anomalies and differences in yield limits of the phases can lead to essential change of pressure in the center of the layer in comparison with its value in absence of phase transition, but under equal external load. The maximal value of external load admitting the considered solution is found. The kinetics of possible isothermal regimes of phase transition leading to change in the time-pressure distribution in the plastic layer is investigated.

**PACS.** 64.70.Kb Solid-solid transitions – 81.30.-t Phase diagrams and microstructures developed by solidification and solid-solid phase transformations

## 1 Introduction

The plane deformation state is realized in long prismatic bodies when the load is normal to the lateral surface and doesn't depend on the $z$-coordinate. The stress and deformation components depend only on $x$ and $y$ coordinates. Tangential stress components are $\tau_{xz} = \tau_{yz} = 0$, because the corresponding shear stresses are absent. When tangential stresses are equal to the yield limit on the boundary of contact, the solution of the states of stress in the plastic layer that is not undergoing a phase transition was obtained in [1] by using the flow theory. In a general case, when tangential stresses are not equal to the yield limit on the boundary of contact, the problem of a compressed plastic layer, which is not undergoing a phase transition, was considered in [2] by using the deformational theory of plasticity. In the case of full developed plastic deformation the results of both papers coincide.

In the present paper, equations for the deformation theory of plasticity for more than one phase with different yield limits and volume jumps are solved. The considered problem is important because the anomalies in the form of "steps" in the pressure distribution were observed experimentally in the high pressure camera with varying geometry as shown in [3] and [4]. The phase transition kinetics is also investigated.

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## 2 Non-linear effects of pressure self-multiplication at phase transition

Let the plastic layer be compressed between flat parallel anvils with the force $2F$ per unit length in the direction and parallel to the axis $y$. The thickness of the layer is $2h$. The layer is thin, so that $\kappa = h/l \ll 1$, where $l$ is half of the layer width. We introduce dimensionless coordinates: $\xi = x/l$, $\eta = y/l$. The stress components satisfy the equations of equilibrium:

$$\frac{\partial \sigma_x}{\partial \xi} + \frac{\partial \tau_{xy}}{\partial \eta} = 0\,, \; \frac{\partial \tau_{xy}}{\partial \xi} + \frac{\partial \sigma_y}{\partial \eta} = 0 \tag{1}$$

and the Mises yield condition

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = 4k^2 \tag{2}$$

where $k$ is yield limit under shear.

It was shown in [2] that in conditions of simple homogeneous loading the results derived with the theory of plastic flows and the deformational theory of plasticity coincide. When the loading becomes complex, these theories can give different results. Application of the theory of plastic flows leads to significant mathematical difficulties which do not give clear results. Because of that, the deformational theory of plasticity is used in the current paper. Let us assume the non-compressibility of a layer in a plastic state

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} = 0 \tag{3}$$

where $u$, $v$ are the displacement components. The equation of the deformational theory of plasticity for a plane deformation is:

$$\frac{2\dfrac{\partial u}{\partial \xi}}{\sigma_x - \sigma_y} = \frac{\dfrac{\partial u}{\partial \eta} + \dfrac{\partial v}{\partial \xi}}{2\tau_{xy}} \, . \quad (4)$$

On the side surface of the layer the boundary condition is:

$$\sigma_x(\xi = 1) = 0 \, . \quad (5)$$

The stress $\sigma_y$ is a static equivalent of the compressing force $2F$:

$$\int_0^1 \sigma_y d\xi = -\frac{F}{l} = -P. \quad (6)$$

At first, we write the known solution of the equations (1-6) for a plastic layer, which is not undergoing a phase transition, as it is necessary for further considerations. Following [2], we can assume for plastic deformation:

$$\tau_{xy} = R(\xi)\frac{\eta}{\kappa} \quad (7)$$

Cross-sections $\eta = 0$, $\eta = \pm\kappa$ remain plane in the process of compression, and therefore $v = v(\eta)$. Then from equation (3) it follows:

$$u = -\frac{\partial v}{\partial \eta}\xi = -v'(\eta)\xi. \quad (8)$$

From (4) and (2) with the help of (8) at $\eta = \kappa$, we find:

$$R(\xi) = -k\frac{C_1\xi}{\sqrt{1 + C_1^2\xi^2}} \quad (9)$$

where

$$C_1 = -\frac{v_1''(\kappa)}{2v_1'(\kappa)} \, .$$

Substituting (7) and (9) in the equation of equilibrium, with the help of (5) we obtain:

$$\sigma_x = \frac{k}{\kappa C_1}\left(\sqrt{1 + C_1^2\xi^2} - \sqrt{1 + C_1^2}\right) . \quad (10)$$

The second equation in (1) and (7) show that $\sigma_y$ depends on $\eta^2$. Since the stress $\sigma_y$ is large and $\eta \ll 1$, we can assume that $\sigma_y$ does not depend on $\eta$. Therefore, in this approximation the second equation in (1) is satisfied automatically. This follows from the detailed discussion in [2]. Then supposing $\eta = \kappa$, we have from (2):

$$\sigma_y = \sigma_x - \frac{2k}{\sqrt{1 + C_1^2\xi^2}} \, . \quad (11)$$

The equation (6) leads to the definition of a constant $C_1$:

$$\frac{1}{4\kappa C_1}\sqrt{1 + C_1^2} + \frac{1}{C_1}\left(1 - \frac{1}{4\kappa C_1}\right) \times \ln\left(C_1 + \sqrt{1 + C_1^2}\right) = \frac{P}{2k} \cdot \quad (12)$$

In this problem as well as in [1] and [2], the boundary conditions for $\tau_{xy}$ are satisfied only in Saint-Venant sense:

$$\int_{-\infty}^{\infty} \tau_{xy}(1, \eta) d\eta = 0 \, .$$

Now we consider a plastic layer that is undergoing two phase transitions, *i.e.* there exist three phases in the layer. Let the yield limit be $k_1$ in a low pressure phase (phase 1), $k_2$ in the intermediate pressure phase (phase 2), and $k_3$ in the high pressure phase (phase 3). The thermodynamic equilibrium transition pressure at a given temperature between phases 1 and 2 is:

$$\sigma_x = -P_1 \quad (13)$$

and between phases 2 and 3:

$$\sigma_x = -P_2 \, . \quad (14)$$

The components of tensor $\sigma_y$ experience discontinuity at the phase boundaries. The locations of thermodynamic equilibrium boundaries between the phases 1 and 2, and the phases 2 and 3 disregarding the volume jumps are respectively $\xi_{10}$ and $\xi_{20}$:

$$\xi_{10} = \frac{\sqrt{\left(\sqrt{1 + C_1^2} - \dfrac{P_1\kappa C_1}{k_1}\right)^2 - 1}}{C_1} \quad (15)$$

$$\xi_{20} = \frac{\sqrt{\left(\sqrt{1 + C_1^2} - \dfrac{P_2\kappa C_1}{k_2}\right)^2 - 1}}{C_1} \, .$$

These formulas were obtained from (10) with the help of (13) and (14).

For given $P$, the value of $C_1$ is determined from (12). Disregarding the volume jumps the region $0 \le \xi \le \xi_{20}$ is occupied by the phase 3, the region $\xi_{20} \le \xi \le \xi_{10}$ by the phase 2, and the region $\xi_{10} \le \xi \le 1$ by the phase 1.

The real values $\xi_{10}$ and $\xi_{20}$, defining a phase boundary at thermodynamic equilibrium with respect to volume jumps at a phase transition, are:

$$\xi_{1n} = \xi_{10}\left[1 - \frac{\delta V_1}{2V_1}\right] \quad (16)$$

$$\xi_{2n} = \xi_{10}\left[1 - \frac{\delta V_2}{2V_2}\right]$$

where $\delta V_1/V_1$ and $\delta V_2/V_2$ are the volume jumps at the phase transitions. Therefore, the phases 1, 2 and 3 will occupy the regions $\xi_{1n} \le \xi \le 1$, $\xi_{2n} \le \xi \le \xi_{1n}$ and $0 \le$

$$\bar{C}_2 = \frac{2(P_2-P_1)\kappa}{k_2\sqrt{\left[\frac{(P_2-P_1)^2\kappa^2}{k_2^2}-(\xi_{2n}-\bar{\xi}_2)^2-(\xi_{1n}-\bar{\xi}_1)^2\right]^2-4(\xi_{1n}-\bar{\xi}_1)^2(\xi_{2n}-\bar{\xi}_2)^2}} \tag{26'}$$

$$\bar{C}_1 = \frac{2P_1\kappa}{k_1\sqrt{\left[(\xi_{1n}-\bar{\xi}_1)^2+(1-\bar{\xi}_1)^2-\frac{{P_1}^2\kappa^2}{k_1^2}\right]^2-4(\xi_{1n}-\bar{\xi}_1)^2(1-\bar{\xi}_1)^2}} \tag{28'}$$

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$\xi \le \xi_{2n}$, respectively. Due to volume jumps, the substance in the phases 1 and 2 will start to flow to both sides of a certain neutral lines $\bar{\xi}_1$ and $\bar{\xi}_2$ . This will cause the pressure behavior anomaly near the phase boundaries.

To determine the pressure-coordinate relations, the system of equations (1–4) in the phases 1, 2, and 3 must be solved with the help of (5) and (6). In the phases 1 and 2, equation (3) gives correspondingly:

$$u_1 = -\bar{v}_1'(\eta)(\xi-\bar{\xi}_1) \tag{17}$$

$$u_2 = -\bar{v}_2'(\eta)(\xi-\bar{\xi}_2)\,. \tag{18}$$

Substituting (17) and (18) in (4) with the help of (3) we define at $\eta=\kappa$ for the phases 1 and 2:

$$R_1 = -k_1\frac{\bar{C}_1\left(\xi-\bar{\xi}_1\right)}{\sqrt{1+\bar{C}_1^2\left(\xi-\bar{\xi}_1\right)^2}} \tag{19}$$

$$R_2 = -k_2\frac{\bar{C}_2\left(\xi-\bar{\xi}_2\right)}{\sqrt{1+\bar{C}_2^2\left(\xi-\bar{\xi}_2\right)^2}} \tag{20}$$

where

$$\bar{C}_1 = -\frac{\bar{v}_1''(\kappa)}{2\bar{v}_1'(\kappa)}\ ,\quad \bar{C}_2 = -\frac{\bar{v}_2''(\kappa)}{2\bar{v}_2'(\kappa)}\,.$$

For the phase 3 the expression (9) is valid with $k=k_3$. On the phase boundary the conditions

$$\sigma_x^{(1)}|_{\xi=\xi_{1n}} = {\sigma_x}^{(2)}|_{\xi=\xi_{1n}} = -P_1 \tag{21}$$

$$\sigma_x^{(2)}|_{\xi=\xi_{2n}} = {\sigma_x}^{(3)}|_{\xi=\xi_{2n}} = -P_2 \tag{22}$$

must be fulfilled. For every phase we assume that equation (7) is valid.

Substituting (7, 19), and (2) in (1) with the help of (5, 21), and (22) we obtain the expression for the stresses in the region $0 \le \xi \le \xi_{2n}$:

$${\sigma_x}^{(3)} = \frac{k_3}{\kappa\bar{C}_3}\left(\sqrt{1+\bar{C}_3^2\xi^2}-\sqrt{1+\bar{C}_3^2\xi_{2n}^2}\right)-P_2 \tag{23}$$

$$\sigma_y^{(3)} = \sigma_x^{(3)} - \frac{2k_3}{\sqrt{1+\bar{C}_3^2\xi^2}} \tag{24}$$

where

$$\bar{C}_3 = -\frac{v_3''(\kappa)}{2v_3'(\kappa)}\ ;$$

in the region $\xi_{2n} \le \xi \le \xi_{1n}$:

$$\sigma_x^{(2)} = \frac{k_2}{\kappa\bar{C}_2} \times\left(\sqrt{1+\bar{C}_2^2\left(\xi-\bar{\xi}_2\right)^2}-\sqrt{1+\bar{C}_2^2\left(\xi_{1n}-\bar{\xi}_2\right)^2}\right)-P_1 \tag{25}$$

$$\sigma_y^{(2)} = \sigma_x^{(2)} - \frac{2k_2}{\sqrt{1+\bar{C}_2^2\left(\xi-\bar{\xi}_2\right)^2}} \tag{26}$$

in the region $\xi_{1n} \le \xi \le 1$:

$$\sigma_x^{(1)} = \frac{k_1}{\kappa\bar{C}_1}\left(\sqrt{1+\bar{C}_1^2\left(\xi-\bar{\xi}_1\right)^2}-\sqrt{1+\bar{C}_1^2\left(1-\bar{\xi}_1\right)^2}\right) \tag{27}$$

$$\sigma_y^{(1)} = {\sigma_x}^{(1)} - \frac{2k_1}{\sqrt{1+\bar{C}_1^2\left(\xi-\bar{\xi}_1\right)^2}} \tag{28}$$

where $\bar{C}_1$ and $\bar{C}_2$ are given by equations (26′) and (28′) above. Combining (17) and (18) with (16) we find

$$\bar{\xi}_1 = \xi_{10}\left(1-\frac{\delta V_1}{2{v'}_1(\eta)V_1}\right)$$
$$\bar{\xi}_2 = \xi_{20}\left(1-\frac{\delta V_2}{2{v'}_2(\eta)V_2}\right)\,. \tag{29}$$

Here ${v'}_1(\eta)$ has the same order of value as ${v'}_2(\eta)$ and as $P/E_k$, where $E_k$ is a local elastic modulus [2].

The unknown constant $C_3$ is defined from the condition (6) that should be written in the form:

$$\int_0^{\xi_{2n}}\sigma_y^{(3)}d\xi+\int_{\xi_{2n}}^{\xi_{1n}}\sigma_y^{(2)}d\xi+\int_{\xi_{1n}}^{1}\sigma_y^{(1)}d\xi=-P\,. \tag{30}$$

Substituting (24, 26), and (28) in (30) we get for $C_3$:

$$\left(-\frac{k_3}{2\kappa\bar{C}_3}\sqrt{1+\bar{C}_3^2{\xi_{2n}}^2}-P_2\right)\xi_{2n}
-\frac{2k_3}{\bar{C}_3}\left(1-\frac{1}{2\kappa\bar{C}_3}\right)\ln\left(\bar{C}_3\xi_{2n}+\sqrt{1+\bar{C}_3^2{\xi_{2n}}^2}\right)
+\frac{k_2}{2\kappa\bar{C}_2}\left(2\xi_{2n}-\xi_{1n}-\bar{\xi}_2\right)\sqrt{1+\bar{C}_2^2(\xi_{1n}-\bar{\xi}_2)^2}
-\frac{k_2}{2\kappa\bar{C}_2}(\xi_{2n}-\bar{\xi}_2)\sqrt{1+\bar{C}_2^2(\xi_{2n}-\bar{\xi}_2)^2}
-\frac{2k_2}{\bar{C}_2}\left(1-\frac{1}{4\kappa\bar{C}_2}\right)\ln\frac{\bar{C}_2(\xi_{1n}-\bar{\xi}_2)+\sqrt{1+\bar{C}_2^2(\xi_{1n}-\bar{\xi}_2)^2}}{\bar{C}_2(\xi_{2n}-\bar{\xi}_2)+\sqrt{1+\bar{C}_2^2(\xi_{2n}-\bar{\xi}_2)^2}}
-P_1(\xi_{1n}-\xi_{2n})+\frac{k_1}{2\kappa\bar{C}_1}(2\xi_{1n}-\bar{\xi}_1-1)\sqrt{1+\bar{C}_1^2(1-\bar{\xi}_1)^2}
-\frac{1}{2\kappa\bar{C}_1}(\xi_{1n}-\bar{\xi}_1)\sqrt{1+\bar{C}_1^2(\xi_{1n}-\bar{\xi}_1)^2}-\frac{2k_1}{\bar{C}_1}\left(1-\frac{1}{4\kappa\bar{C}_1}\right)
\times\ln\frac{\bar{C}_1(1-\bar{\xi}_1)+\sqrt{1+\bar{C}_1^2(1-\bar{\xi}_1)^2}}{\bar{C}_1(\xi_{1n}-\bar{\xi}_1)+\sqrt{1+\bar{C}_1^2(\xi_{1n}-\bar{\xi}_1)^2}}=-P. \tag{31}$$

The solution of (23-28) and (31) is metastable since the expressions (25, 28) have maximum in the region of phase boundaries, *i.e.* $-\sigma_{x,m}^{(1)} > P_1$, $-\sigma_{x,m}^{(2)} > P_2$. The regions of metastability are:

$$\Delta_1 = 2(\xi_{m,1}-\xi_{1n}) = 2\left(\xi_{m,1}-\xi_{10}+\frac{\xi_{10}}{2}\frac{\delta V_1}{V_1}\right)$$
$$\Delta_2 = 2(\xi_{m,2}-\xi_{2n}) = 2\left(\xi_{m,2}-\xi_{20}+\frac{\xi_{20}}{2}\frac{\delta V_2}{V_2}\right)$$

where $\xi_{m,1}$ and $\xi_{m,2}$ are defined from conditions, correspondingly:

$$\frac{\partial\sigma_x^{(1)}}{\partial\xi}|_{\xi=\xi_{m,1}}=0\ \wedge\ \frac{\partial\sigma_x^{(2)}}{\partial\xi}|_{\xi=\xi_{m,2}}=0.$$

That gives $\xi_{m,1}=\bar{\xi}_1$ and $\xi_{m,2}=\bar{\xi}_2$. Therefore with the help of (29) we get:

$$\Delta_1 = 2(\bar{\xi}_1-\xi_{1n}) = \xi_{10}\left(1-\frac{1}{v'_1(\eta)}\right)\frac{\delta V_1}{V_1}$$
$$\Delta_2 = 2(\bar{\xi}_2-\xi_{2n}) = \xi_{20}\left(1-\frac{1}{v'_2(\eta)}\right)\frac{\delta V_2}{V_2}. \tag{32}$$

During the relaxation of the substance in the region of metastable state there form the regions of a phase mixture with the dimensions $\Delta_1$ and $\Delta_2$ at the pressures $P_1$ and $P_2$ , *i.e.* in the regions of the phase boundaries a "step" is formed. The stable solution of the system (1-6) in the region $0\le\xi\le\xi_{2n}$ is:

$$\sigma_x^{(3)}=\frac{k_3}{\kappa\bar{C}_3}\left(\sqrt{1+\bar{C}_3^2\xi^2}-\sqrt{1+\bar{C}_3^2{\xi_{2n}}^2}\right)-P_2$$
$$\sigma_y^{(3)}=\sigma_x^{(3)}-\frac{2k_3}{\sqrt{1+\bar{C}_3^2\xi^2}}. \tag{33}$$

In the region of phase mixture, $\xi_{2n}\le\xi\le\xi_{2*}$ where $\xi_{2*}=\xi_{2n}+\Delta_2$

$$\sigma_x=-P_2. \tag{34}$$

In this region compressibility is very large and equation (3) is not valid. As follows from the Mises yield condition (2), $\sigma_y$ in the region of phase mixtures differs from $\sigma_x=-P_2$ by the value that has an order of effective yield limit $k_{eff}$ for phase mixtures. However, it is known from [6] that due to phase dislocations the atoms become highly mobile at the phase boundaries which significantly reduces $k_{eff}$ and, hence reduces also the effective value $\tau_{xy}$. In that case, as follows from (2), $\sigma_y\approx\sigma_x=-P_2$; in the region $\xi_{2*}\le\xi\le\xi_{1n}$:

$$\sigma_x^{(2)}=\frac{k_2}{\kappa\bar{C}_2}\left(\sqrt{1+\bar{C}_2^2\xi^2}-\sqrt{1+\bar{C}_2^2\xi_{1n}^2}\right)-P_1$$
$$\sigma_y^{(2)}={\sigma_x}^{(2)}-\frac{2k_2}{\sqrt{1+\bar{C}_2^2\xi^2}} \tag{35}$$

in the region $\xi_{1n}\le\xi\le\xi_{1*}$, where $\xi_{1*}=\xi_{1n}+\Delta_1$

$$\sigma_x=-P_1$$
$$\sigma_y\simeq\sigma_x \tag{36}$$

and, finally, in the region $\xi_{1*}\le\xi\le 1$

$$\sigma_x^{(1)}=-\frac{k_1}{\kappa\bar{C}_1}\left(\sqrt{1+\bar{C}_1^2\xi^2}-\sqrt{1+\bar{C}_1^2}\right)$$
$$\sigma_y^{(1)}={\sigma_x}^{(1)}-\frac{2k_1}{\sqrt{1+\bar{C}_1^2\xi^2}} \tag{37}$$

where

$$\bar{C}_2=\frac{2(P_2-P_1)\kappa}{k_2\sqrt{\left(\xi_{2*}^2+\xi_{1n}^2-\frac{(P_1-P_2)^2\kappa^2}{k_2^2}\right)^2-4\xi_{2*}^2\xi_{1n}^2}}$$
$$\bar{C}_1=\frac{2P_1\kappa}{k_1\sqrt{\left(\xi_{1*}^2+1-\frac{P_1^2\kappa^2}{k_1^2}\right)^2-4\xi_{1*}^2}}.$$

In this case the condition (6) has the form

$$\int_0^{\xi_{2n}}\sigma_y^{(3)}d\xi+\int_{\xi_{2*}}^{\xi_{1n}}\sigma_y^{(2)}d\xi+\int_{\xi_{1*}}^{1}\sigma_y^{(1)}d\xi
=-P+P_1(\xi_{1*}-\xi_{1n})+P_2(\xi_{2*}-\xi_{2n}). \tag{38}$$

Substituting (33, 35), and (37) in (38) we obtain the equation for the constant $C_3$:

$$\begin{aligned}
&\frac{2k_3}{\bar{C}_3}\left(\frac{1}{4\kappa\bar{C}_3}-1\right)\ln\left(\bar{C}_3\xi_{2n}+\sqrt{1+\bar{C}_3^2\xi_{2n}^2}\right)\\
&-\frac{k_3}{2\kappa\bar{C}_3}\sqrt{1+\bar{C}_3^2\xi_{2n}^2}\,\xi_{2n}\\
&+\frac{2k_2}{\bar{C}_2}\left(\frac{1}{4\kappa\bar{C}_2}-1\right)\ln\frac{\bar{C}_2\xi_{1n}+\sqrt{1+\bar{C}_2^2\xi_{1n}^2}}{\bar{C}_2\xi_{2*}+\sqrt{1+\bar{C}_2^2\xi_{2*}^2}}\\
&+\frac{k_2}{2\kappa\bar{C}_2}\sqrt{1+\bar{C}_2^2\xi_{1n}^2}(2\xi_{2*}-\xi_{1n})\\
&-\frac{k_2}{2\kappa\bar{C}_2}\sqrt{1+\bar{C}_2^2\xi_{2*}^2}\,\xi_{2*}\\
&+\frac{2k_1}{\bar{C}_1}\left(\frac{1}{4\kappa\bar{C}_1}-1\right)\ln\frac{\bar{C}_1+\sqrt{1+\bar{C}_1^2}}{\bar{C}_1\xi_{1*}+\sqrt{1+\bar{C}_1^2\xi_{1*}^2}}\\
&+\frac{k_1}{2\kappa\bar{C}_1}\left(\sqrt{1+\bar{C}_1^2}(2\xi_{1*}-1)-\xi_{1*}\sqrt{1+\bar{C}_1^2\xi_{1*}^2}\right)\\
&=-P+P_1(\xi_{1*}-\xi_{2*})+P_2\xi_{2*}\,.
\end{aligned}\qquad(39)$$

These results are realized in the case of two independent phase transitions. However, it is possible to observe two successive phase transitions, when the first transition initiates the second. This will happen in the case when after the first transition the pressure in the center of the layer becomes higher than the thermodynamic equilibrium pressure of the second transition.

For the solution of the problem of two successive transitions it is necessary to get a metastable and a stable solution at one transition with thermodynamic equilibrium pressure $P_1$ and volume jump $\delta V_1/V_1$. In this case the metastable solution defining the unknown constant $C_{21}$ has the form:

$$\begin{aligned}
&\left(-\frac{k_2}{2\kappa\bar{C}_{21}}\sqrt{1+\bar{C}_{21}^2\xi_{1n}^2}-P_1\right)\xi_{1n}\\
&-\frac{2k_2}{\bar{C}_{21}}\left(1-\frac{1}{2\kappa\bar{C}_{21}}\right)\ln\left(\bar{C}_{21}\xi_{1n}+\sqrt{1+\bar{C}_{21}^2\xi_{1n}^2}\right)\\
&+\frac{k_1}{2\kappa\bar{C}_1}(2\xi_{1n}-\bar{\xi}_1-1)\sqrt{1+\bar{C}_1^2(1-\bar{\xi}_1)^2}\\
&-\frac{k_1}{2\kappa\bar{C}_1}(\xi_{1n}-\bar{\xi}_1)\sqrt{1+\bar{C}_1^2(\xi_{1n}-\bar{\xi}_1)^2}\\
&-2\frac{k_1}{\bar{C}_1}\left(1-\frac{1}{4\kappa\bar{C}_1}\right)\ln\frac{\bar{C}_1(1-\bar{\xi}_1)+\sqrt{1+\bar{C}_1^2(1-\bar{\xi}_1)^2}}{\bar{C}_1(\xi_{1n}-\bar{\xi}_1)+\sqrt{1+\bar{C}_1^2(\xi_{1n}-\bar{\xi}_1)^2}}\\
&=-P\,.
\end{aligned}\qquad(40)$$

For the stable solution we get:

$$\begin{aligned}
&\frac{k_2}{2\kappa\bar{C}_{21}^2}\ln\left(\bar{C}_{21}\xi_2+\sqrt{1+\bar{C}_{21}^2\xi_2^2}\right)\\
&-\frac{2k_2}{\bar{C}_{21}}\ln\left(\bar{C}_{21}\xi_2+\sqrt{1+\bar{C}_{21}^2\xi_2^2}\right)\\
&-\frac{k_2}{2\kappa\bar{C}_{21}}\xi_2\sqrt{1+\bar{C}_{21}^2\xi_2^2}\\
&-\frac{2k_1}{\bar{C}_1}\ln\frac{\bar{C}_1+\sqrt{1+\bar{C}_1^2}}{\bar{C}_1\xi_{1*}+\sqrt{1+\bar{C}_1^2\xi_{1*}^2}}-\frac{k_1}{\kappa\bar{C}_1}(1-\xi_{1*})\sqrt{1+\bar{C}_1^2}\\
&+\frac{k_1}{2\kappa\bar{C}_1^2}\ln\frac{\bar{C}_1+\sqrt{1+\bar{C}_1^2}}{\bar{C}_1\xi_{1*}+\sqrt{1+\bar{C}_1^2\xi_{1*}^2}}\\
&+\frac{k_1}{2\kappa\bar{C}_1}\left(\sqrt{1+\bar{C}_1^2}-\xi_{1*}\sqrt{1+\bar{C}_1^2\xi_{1*}^2}\right)\\
&=-P+P_1\xi_{1*}\,.
\end{aligned}\qquad(41)$$

The location of equilibrium boundary $\xi_{2n}$ between phases 2 and 3 at successive transitions is:

$$\xi_{2n}=\xi_{20}\left(1-\frac{\delta V_2}{2V_2}\right)\qquad(42)$$

where

$$\xi_{20}=\left[\sqrt{\left(\sqrt{1+\bar{C}_{21}^2\xi_{1n}^2}-\frac{(P_2-P_1)\kappa\bar{C}_{21}}{k_2}\right)^2-1}\right]/\bar{C}_{21}\,.$$

The constants $C_2$ and $C_3$ for metastable and stable solutions are defined from the same formulas as for two independent transitions with the help of (42).

For the first example we consider two successive phase transitions in the plastic layer without strain hardening $P/Ek\gg 1$. We have $P=3$ GPa, $\kappa=10^{-2}$, yield limits of the substance of the layer $k_1=k_2=k_3=0.3$ GPa, and the volume jumps at transitions $\delta V_1/V_1=\delta V_2/V_2=0.2$. We assume the values of thermodynamic equilibrium transition pressure to be $P_1=1$ GPa and $P_2=3.8$ GPa. The maximum pressure value in the layer which is not undergoing a phase transition is $-\sigma_{x,m}^{(1)}=3.6$ GPa. Since $P_2>-\sigma_{x,m}^{(1)}$, two independent transitions cannot take place in the system. With the help of the stable solution (41) received above, we get after the first transition $-\sigma_{x,m}^{(2)}=4.05$ Gpa, and after the second $-\sigma_{x,m}^{(3)}=4.15$ GPa. The distributions of pressure with coordinate $\xi$ is shown in Figure 1. One can see that at unchanged external load $P$, two successive transitions in the layer cause the pressure self-multiplication or the positive feedback. The pressure causes the phase transition, after that the value of pressure in the center of the layer increases causing the transition with the pressure increasing in the center.

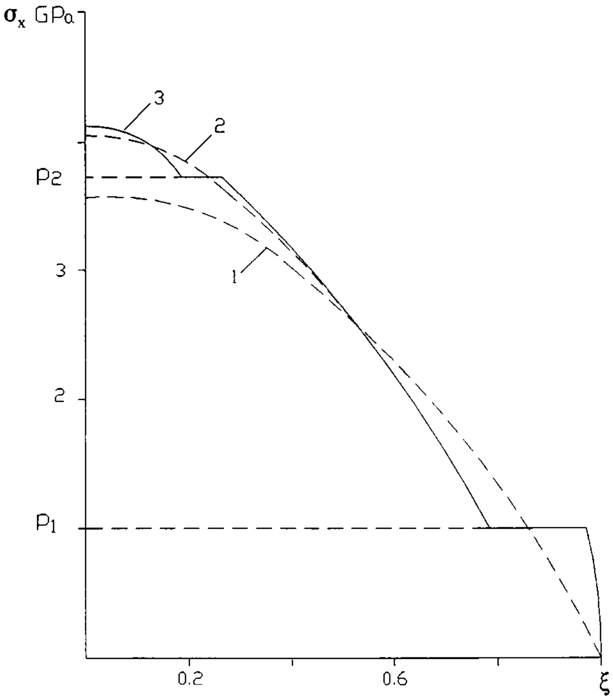


**Fig. 1.** The distribution of the pressure with coordinate $\xi$ at $P = 3$ GPa, $\kappa = 10^{-2}$, $k_1 = k_2 = k_3 = 0.3$ GPa, and $\delta_1/V_1 = \delta_2/V_2 = 0.2$.

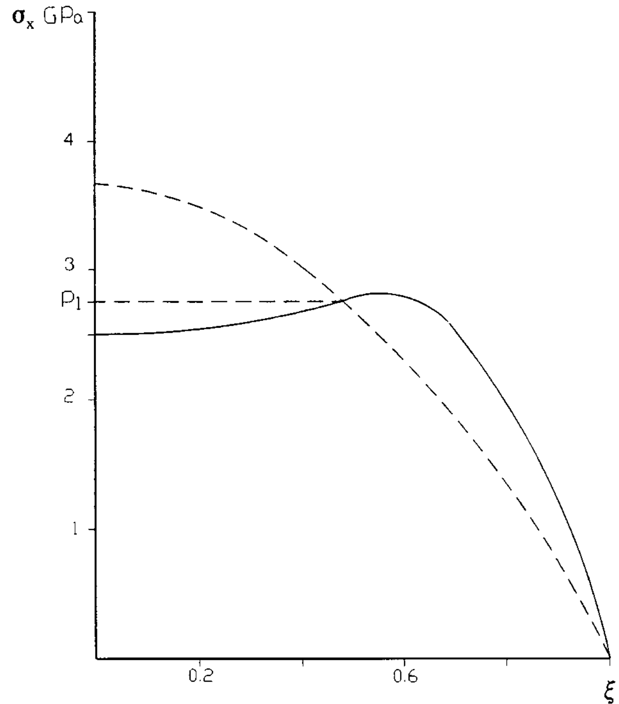


**Fig. 3.** The pressure distribution for the metastable solution at $P = 3$ GPa, $\kappa = 10^{-2}$, $\delta_1/V_1 = 0.1$, $P_1 = 2.7$ GPa, $k_1 = k_2 = 0.3$ GPa.

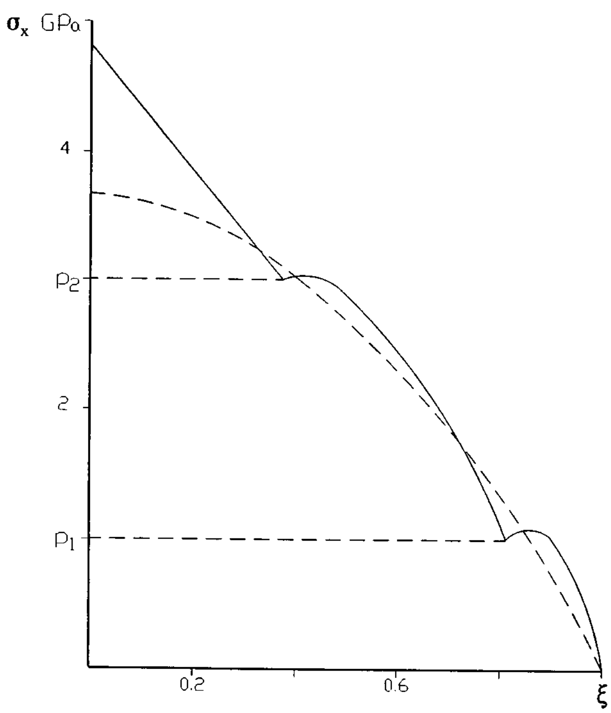


**Fig. 2.** The pressure distribution for the metastable solution of two independent phase transitions at $P = 3$ GPa, $\kappa = 10^{-2}$, $k_1 = k_2 = 0.3$ GPa, $k_3 = 0.05$ GPa, $P_1 = 1$ GPa, $P_2 = 3$ GPa, $\delta_1/V_1 = \delta_2/V_2 = 0.1$.

The pressure distribution for the metastable solution of two independent phase transitions is shown in Figure 2 for following conditions $P = 3$ GPa, $\kappa = 10^{-2}$, $k_1 = k_2 = 0.3$ GPa, $k_3 = 0.05$ GPa, $P_1 = 1$ GPa, $P_2 = 3$ GPa, $\delta V_1/V_1 = \delta V_2/V_2 = 0.1$. In this case the value of maximum pressure is $-\sigma^{(3)}_{x,m} = 4.855$ GPa, *i.e.* the pressure self multiplication is observed. It is interesting that at one phase transition in the layer at yield limit $k = 0.5$ GPa and $P = 3$ GPa, $\kappa = 10^{-2}$ the solution does not exist, so the phases 1 and 2 at greater yield limits restrain the flowing of the matter existing at the phase 3 at low yield limit.

However, the phase transition in the layer does not always result in the pressure self-multiplication in the center of the layer. For example, in Figure 3 the metastable solution is shown for phase transition at $P = 3$ GPa, $\kappa = 10^{-2}$, $\delta V_1/V_1 = 0.1$, $P_1 = 2.7$ GPa, $k_1 = k_2 = 0.3$ GPa. In this case the pressure in the center of the layer is near 2.5 GPa, *i.e.* less than the equilibrium transition pressure. In this case, the phase transition in the system did not realize.

At $k_3 \geq k_2 \geq k_1$ the pressure self-demultiplication is possible too. At a given $P$, the pressure in the center of the layer at metastable state is less than at stable state.

It must be noted that the obtained above solution exists when the square root expressions in definitions of constants $\bar{C}_1$ and $\bar{C}_2$ exceed zero. This condition lead to inequality:

$$\frac{1}{\bar{C}_1}\sqrt{\left(\sqrt{1+\bar{C}_1^2}-\frac{\kappa\bar{C}_1P_1}{k_1}\right)^2-1}\left(1+\frac{\delta V_1}{2V_1}\right)<1-\frac{P_1\kappa}{k_1}$$

where $\bar{C}_1$ is the solution of (12). The inequality means, that at given parameters $P_1$, $\kappa$ , $k_1$ and $\delta V_1/V_1$ maximum load $P$ exists, above which the considered states do not realize.

Finally it is necessary to make some notes. The solution for the mixed phases describes the final state of mechanical and thermodynamic equilibrium. This state is defined by the phase diagram and the value of volume jump and does not depend on the mechanism of phase transition.

The solution with the local maximum of the pressure distribution describes mechanical equilibrium but the thermodynamic equilibrium is not achieved. Therefore metastable states may be realized at corresponding scenarios of phase transition and following relaxation to the thermodynamic equilibrium state.

The anomalies in the form of "steps" in the pressure distribution was experimentally observed in potassium chloride transition in the system of parallel disks [3], and for the case of two transition in chalcogenides (for example, PbTe and PbSe) in the system of rounded cone-planes [4]. Therefore, the arising of discussed anomalies doesn't depend on the geometry of the high pressure camera.

## 3 Kinetics of isothermal regime of phase transition

This chapter considers some possible scenarios of isothermal regimes of phase boundary motion. We suppose that the phase boundary motion happens at quasi-equilibrium and the system is at the mechanical equilibrium.

The condition that the latent heat of transition does not influence the velocity of the stable phase growth or the condition of isothermal regime of transition is $U(\tau_*/\chi)^{1/2}\ll 1$. Here $U$ is the velocity of the phase boundary motion normal to the boundary of division; $\tau_*$ is the time of the thermal relaxation in the layer; and $\chi$ is the thermal diffusivity of the layer. We find the dependence of the stable phase growth velocity and of pressure distribution with time when the system relaxes to the thermodynamic equilibrium state. The kinetics of phase transition on the boundaries of the first and second phases can be expressed as in [7]:

$$U=2Se^{\left(-\frac{E+\Delta\bar{H}}{T_0}\right)}\sinh\left(-\frac{\sigma_{x,1}+P_1}{2T_0}\delta V_1\right) \tag{43}$$

where $\Delta H$ is the heat transition per one particle (expressed in degrees) at the temperature $T=T_0$; $\Delta\bar{H}$ is equal to $\Delta H+\delta V_1P_1/2$; $E$ is activation energy of transition (expressed in degrees); $S$ is a coefficient at the exponential term; $\sigma_{x,1}$ is the pressure on the phase boundary at the transition process; $V_1$ and $V_2$ are volumes of the first and second phases per one particle, and $\delta V_1=V_1-V_2$ is the difference of these volumes.

The expression (43) is based on the theory of the absolute reaction velocity [8]. It allows to calculate the particle flow through the phase boundary with the average potential barrier. Considering for simplicity, the kinetics of transformation with one transition the following conditions exist:

$$\xi_{1n}=\xi_{2n}\,,\ P_1=P_2\,,\ k_3=k_2\,,\ \bar{C}_3=\bar{C}_2\,,\ \bar{\xi}_1=\bar{\xi}_2\,. \tag{44}$$

In accordance with symmetry of the problem, the source of growth is the stable phase region of the volume $2\kappa l^2\Delta\xi^0$, where $\Delta\xi^0=|\xi_1^0-\xi_2^0|$. When the source is in the center of the layer the high pressure phase of dimension $\xi_1\ll 1$ is formed. In case of disregarding the volume jump the position of the phase boundary at a given moment of time is designated $\xi_{10}$. Then the boundary position with respect to a volume jump is

$$\xi_1=\left(1-\frac{1}{2}\frac{\delta V_1}{V_1}\right)\xi_{10}\,.$$

With the help of (10) and (12) at $k=k_1$ and $C_1\ll 1$ the pressure on the phase boundary is

$$\sigma_{x,1}=\frac{3}{2}(P-2k_1)\left[\xi_1^2\left(1+\frac{\delta V_1}{V_1}\right)-1\right]. \tag{45}$$

$$\sigma_x|_{\xi=\xi_{1n}}=-P_1=\frac{3}{2}(P-2k_1)\left[\xi_{1n}^2\left(1+\frac{\delta V_1}{V_1}\right)-1\right]. \tag{46}$$

Substituting (45) and (46) in (43), we obtain the equation for the time dependence $\xi_1=\xi_1(t)$ of the phase boundary moving away from the center of the layer. Analytical results can be obtained assuming $(\sigma_{x,1}+P_1)\delta V1/2T_0\ll 1$. For such a case equation derived with the help of (45) and (46) becomes:

$$\frac{dx}{d\tau}=1-x^2 \tag{47}$$

where

$$x=\frac{\xi_1}{\xi_{1n}}\,,\ \tau=\gamma t$$

$$\gamma=\frac{3S}{2l}\frac{(P-2k_1)}{T_0}\delta V_1\xi_{1n}e^{-\frac{E+\Delta\bar{H}}{T_0}}\,.$$

The solution (47) has a form:

$$\xi_1=\xi_{1n}\frac{me^{2\tau}-1}{me^{2\tau}+1} \tag{48}$$

where

$$m=\frac{\left(1+\frac{\xi_1^0}{\xi_{1n}}\right)}{\left(1-\frac{\xi_1^0}{\xi_{1n}}\right)}\,,\ \frac{\xi_1^0}{\xi_{1n}}<1.$$

Substituting (48) and (45) in (31) and (27) instead of $\xi_{1n}$ and $P_1$ with respect to (44, 29), and (16) we obtain from (23) and (27) the time pressure distribution when the system relaxes to the state of thermodynamic equilibrium. It is important to emphasize here that the studied problem is statically determinate during the process of phase transition. The movements of phase boundaries happen without matter flow from the region between the anvils. Hence, the derived solution permits an existence of a layer with a constant thickness $\kappa$ . The derived solution showed that the values of $u$ and $v$ and their derivatives vary with time. This means that for $u, v \ll h$, the change of the thickness of the layer can be neglected in a partial phase transition. Otherwise, the problem becomes statically indeterminate, and it should be considered using kinematic conditions as in [1].

The time-pressure distribution of a statistically determinate process is shown in Figure 4. The pressure in the center increases in time. However, the forming of the source of growth in the center of the layer is not always thermodynamic advantageous, so it is possible that the pressure in the center falls lower than the thermodynamic equilibrium pressure.

Suppose for simplicity $k_1 = k_2 = k_3$. Then it follows from (43) that the motion of the phase boundary is defined by the solution of the equation of Rikkati:

$$\frac{d\xi_2}{d\tau} = -\phi(\tau) + \psi(\tau)\xi_2^2 \tag{49}$$

where

$$\phi(\tau) = \frac{3}{2}\alpha\frac{(P-2k)}{P_1}\left[1 - \left(1 + \frac{\delta V_1}{V_0}\right)\xi_1^2(\tau)\right] - \alpha + \psi(\tau)\xi_1^2(\tau)$$

$$\begin{aligned}\psi(\tau) = {} & \frac{3}{2}\alpha\frac{\xi_1^{-3}(\tau)}{P_1}\left[P - (2k - \sigma_{x,1})\xi_1(\tau)\right. \\ & - \frac{k}{2\kappa\bar{C}_1(\tau)}\sqrt{1 + \bar{C}_1^2\left(1 - \bar{\xi}_1(\tau)\right)^2} \\ & \times \left[1 + \bar{\xi}_1(\tau) - 2\xi_1(\tau)\right] \\ & - \frac{k}{2\kappa\bar{C}_1(\tau)}\left(\xi_1(\tau) - \bar{\xi}_1(\tau)\right)\sqrt{1 + \bar{C}_1^2(\tau)\left(\xi_1(\tau) - \bar{\xi}_1(\tau)\right)^2} \\ & + \left(\frac{k}{2\bar{C}_1^2(\tau)} - \frac{2k}{\bar{C}_1(\tau)}\right) \\ & \times \ln\frac{\bar{C}_1(\tau)\left(1 - \bar{\xi}_1(\tau)\right) + \sqrt{1 + \bar{C}_1^2(\tau)\left(1 - \bar{\xi}_1(\tau)\right)^2}}{\bar{C}_1(\tau)\left(\xi_1(\tau) - \bar{\xi}(\tau)\right) + \sqrt{1 + \bar{C}_1^2(\tau)\left(\xi_1(\tau) - \bar{\xi}_1(\tau)\right)^2}}\end{aligned}$$

$$\alpha = \frac{1 - \left(1 + \frac{\delta V_1}{V_1}\right)\xi_{1n}^2}{\left(1 + \frac{\delta V_1}{V_1}\right)\xi_{10}^2}$$

and $\xi_2$ is the phase boundary moving to the center.

Equation (49) can be integrated only approximately. For example, at $P_1 = 2$ GPa, $P = 3$ GPa; $\delta V_1/V_1 = 0.1$; $\xi_{\rm ln} = 0.635$; $\kappa = 10^{-2}$ and ${\xi_2}^0 = 0.43$ the motion velocity of the phase boundary $\xi_2$ can be presented as a series of $\tau$.

$$\frac{d\xi_2}{d\tau} = -0.367 + 0.11\tau - 2.18\tau^2 + 5.44\tau^3 \,. \tag{50}$$

The solution (50) is valid at $\tau \ll 0.4$ and it has a maximum at $\tau = 0.03$. The change of the pressure distribution changes the velocity of phase boundary $\xi_2$. At $\xi_1^0 \approx \xi_2^0 \approx \xi_{1n}$,

$$\phi(\tau) = \psi(\tau)\xi_{1n}^2 \approx C\xi_{1n}^2$$

where $C$ is constant. From (49) it follows at $C > 0$:

$$\xi_2 = \xi_{1n}\frac{m_1 e^{-2\tau} - 1}{me^{-2\tau} + 1}\,, \tag{51}$$

where

$$m_1 = \frac{\left(1 + \frac{\xi_2^0}{\xi_{1n}^0}\right)}{\left(1 - \frac{\xi_2^0}{\xi_{1n}^0}\right)}\,,\ \frac{\xi_2^0}{\xi_{1n}} < 1$$

$$\tau = \alpha\gamma C\xi_{1n}t\,.$$

The final velocity and the time of the complete transition is:

$$\frac{d\xi_2}{d\tau}|_{\tau_n} = -\xi_{1n}\,,\ \tau_n = \frac{1}{2}\ln(m_1)\,.$$

On the reverse run, at the decrease of the loading, the periphery part of the high pressure phase turns out to be in metastable state. Therefore, the phase boundary will move directly to the center of the layer. In this case:

$$\xi_1 = \xi_{1n}\frac{\tilde{m}_1 e^{2\tau} - 1}{\tilde{m}_1 e^{2\tau} + 1} \tag{52}$$

$$\tilde{m}_1 = \frac{\left(1 + \frac{\xi_1^0}{\xi_{1n}}\right)}{\left(1 - \frac{\xi_1^0}{\xi_{1n}}\right)}\ ;$$

$$\tau = \frac{S}{T_0 l}e^{\left(-\frac{E + \Delta H}{T_0}\right)}\delta V_1\xi_{1n}P_1\psi(\xi_1^0)t,$$

where $\xi_1^0$ is the initial location of the phase boundary; $\xi_{1n}$ is the location of the thermodynamic equilibrium phase boundary at the given value of $\tau$ during the reverse run; $\xi_1^0/\xi_{1n} > 1$.

In the center of the layer, the motion of the phase boundary described with equation (52) causes the pressure decrease in time.

## 4 Investigation of the motion stability of the phase transition front

The possibility of kinetic regimes realization considered above depends on the morphological stability of the phase boundary, *i.e.* on the stability of phase transition front to distortion.

In the moment $t_0$ the vector $\bar{\xi} = \bar{\xi}(\tau 0)$ defines the location of the plane phase boundary. The accidental plane front distortion results in the position of the phase boundary in the moment $t_0 + \delta t$:

$$\overline{r} = \bar{\xi}(\tau_0) + Ul^{-1}n\delta t\,. \tag{53}$$

where $\mathbf{n}$ is the unit vector, directed normally to the phase boundary. The instability development in time is defined by equation:

$$r = \bar{\xi}(\tau_0) + l^{-1}\int_{t_0}^{t} Undt\,. \tag{54}$$

The equation describing the evolution of the vector $\mathbf{n}$ was obtained in [9] where it was used for the analysis of positions of layers in superconductors in intermediate state:

$$\frac{\partial n}{\partial t} + U(n\nabla)n = -\nabla U + n(n\nabla U)\,. \tag{55}$$

In quasi-stationary approximation

$$\frac{\partial n}{\partial t} = 0\,,$$

we suppose that all values depend on the coordinate $\xi$ . Then the equation (55) can be written as:

$$\begin{aligned} Un_x\frac{dn_x}{d\xi} &= -\frac{dU}{d\xi} + n_x^2\frac{dU}{d\xi} \\ U\frac{dn_z}{d\xi} &= n_z\frac{dU}{d\xi}\,, \end{aligned} \tag{56}$$

where coordinate $z$ is parallel to the undisturbed phase boundary. Equations (56) and (43) in the approximation $(\sigma_{x,1} + P_1)\delta V_1/2T_0 \ll 1$ have the following solutions. For $n_y = 0$ and $n_x^2 + n_z^2 = 1$ for the moving of the phase boundary from the center:

$$n_z = n_z^0 \frac{\left[1 - \left(\dfrac{\xi_1}{\xi_{1n}}\right)^2\right]}{\left[1 - \left(\dfrac{\xi_1(\tau_0)}{\xi_{1n}}\right)^2\right]} \tag{57}$$

and for the moving of the phase boundary to the center:

$$n_z = n_z^0 \frac{\left[1 - \left(\dfrac{\xi_2}{\xi_{1n}}\right)^2\right]}{\left[1 - \left(\dfrac{\xi_2(\tau_0)}{\xi_{1n}}\right)^2\right]} \tag{58}$$

where $n_z^0$ is the projection of the vector $n$ on $z$-axis in an arbitrary point of distorted phase boundary in the moment $t_0$.

Substituting (57) in (54) with the help of (48) and (58) in (54) with the help of (51) we obtain the expressions describing the time variations of small distortions ($n_z \ll 1$) for the phase boundary moving from the center and to the center, respectively. The influence of surface tension on morphological stability is not taken into consideration here.

Equation (57) shows that for the negative feedback the phase boundary motions from the center of the layer are morphologically stable. As follows from (58) in case of the positive feedback, the phase boundary motions to the center of the layer are morphologically unstable. However, this morphological instability does not cause changes of radius-pressure distribution. The phase boundary motion to the center of the layer on the reverse run is morphologically stable.

## 5 Conclusion

Starting from a well-known classical problem of pressure distribution in homogeneous elastic-plastic plate [2], the current paper considers several problems of pressure distribution in such a plate during phase transitions and the kinetics of these transitions. The small perturbation formalism was used and the following estimation of the relative magnitude of elastic and plastic deformations was performed. The elastic deformation, $u_e/l$, is approximately equal to $\mu k\xi/E$, where $\mu$ is the Poisson's coefficient and $E$ is the ordinary Young's modulus. The step in the pressure distribution (Fig. 1) is caused mostly by the plastic deformation, $u_p/l$, which is about $\Delta/2$. Therefore, $u_p/u_e \approx (E/2\nu k)(\delta V/V)$. For example, when $E = 30$ GPa, $k = 0.3$ GPa, $\mu = 0.3$, and $\delta V/V \approx 0.1$, the ratio $u_p/u_e \approx 16.7$. That means that the plastic deformation is at least an order of the value more than elastic deformation. In case of $u_p/u_e \ll 1$, uniaxial state of pressure exists in the system which is characterized by $\sigma_z \neq 0$ and $\sigma_x$ approaching 0. That makes impossible the movement of the phase boundaries as follows from (43). On the other hand, if the plastic flow governs the phase transition, it is impossible to derive the solutions as presented in the current paper.

Analysis shows that for the considered ranges of values of phase transition pressure, layer thickness, yield limit, and volume jumps, there is a maximum external loading above which the derived solutions do not exist.

Particle velocity on the phase boundary can be defined as $w = U\delta V/V$. When $U = 0.001$ m/s and $\delta V/V = 0.1$, $w$ is about $10^{-4}$ m/s. At such small velocities, it reasonable to analyze phase transitions based on the deformational theory of plasticity.

From simultaneously solved (10) and (12) follows that for $\kappa \ll 1$, the distribution of $\sigma_x$ and $\sigma_y$ is practically independent of $\kappa$. Physically this means that when slow flow of plastic material from the zone between anvils occurs ($\kappa$ diminishes) at constant external loading, $\sigma_x$ and $\sigma_y$ do not change. When $\kappa$ decreases due to the volume jump at phase transitions, the distribution of $\sigma_x$ and $\sigma_y$ remains unchanged for a statically determinate problem.

Changes in $\sigma_x$ and $\sigma_y$ distributions at phase transitions are caused by changing ratios of the second derivatives of displacement $v$ in respect to coordinate to their first

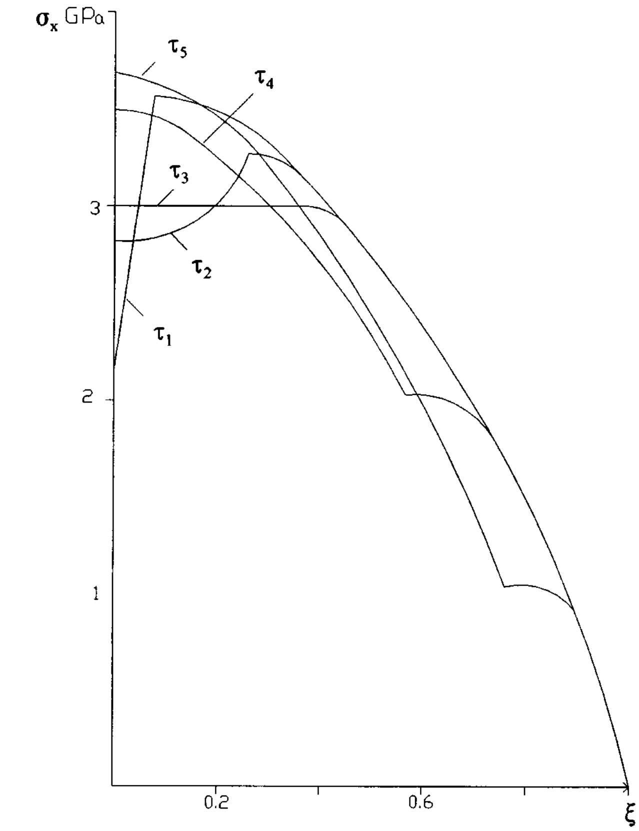


**Fig. 4.** The time-pressure distribution when the system relaxes to the state of thermodynamic equilibrium; $\tau_1 < \tau_2 < \tau_3 < \tau_4 < \tau_5$.

derivatives:

$$\bar{C}_1 = -\frac{\bar{v''}_1(\kappa)}{2v'_1(\kappa)}\,, \bar{C}_2 = -\frac{\bar{v''}_2(\kappa)}{2v'_2(\kappa)}\,, \text{and} \;\; \bar{C}_3 = -\frac{\bar{v''}_3(\kappa)}{2v'_3(\kappa)} \cdot$$

As a result of that, when the volume jump happens at the pressure much smaller than the external loading, $P$, the pressure in the middle increases (Fig. 1). When the phase transition occurs at the pressure similar to the external loading, the pressure in the middle decreases following the volume jump (Fig. 4). For the small yield limit of the phase, large loading, and small volume jump (Fig. 2) the kinetic realization of the state of stresses does not go through the stage of local pressure drop (Fig. 4). The kinetics in (51) does not produce the local pressure drop as well.

Non-linear effect of pressure self-multiplication is positive feedback and is similar to the effect of self-focusing of waves [10]. The results of this paper are physically reasonable and mathematically correct. However, the existence of the suggested kinetic solutions can be proved only experimentally.

## 6 List of symbols

| | |
|---|---|
| $E$ | the ordinary Young's modulus; |
| $F$ | half of the force per unit length applied to the layer in the direction $z$ [N/m]; |
| $h$ | half of the thickness of the layer [m]; |
| $k$ | yield limit under shear [Pa]; |
| $l$ | half of the layer width [m]; |
| $\mathbf{n}$ | the unit vector directed normally to the phase boundary; |
| $P$ | pressure [Pa]; |
| $P_1$, $P_2$ | equilibrium transition pressure between phases 1 and 2; 2 and 3 [Pa]; |
| $u$, $v$ | displacement components [m]; |
| $\Delta i$ | width of pressure step at phase translation; |
| $\xi$ | $x/l$ dimensionless coordinate; |
| $\xi_{10}$, $\xi_{20}$ | the location of thermodynamic equilibrium boundary between the phases 1 and 2; 2 and 3 when the volume jump is disregarded; |
| $\xi_1^0$, $\xi_2^0$ | the initial locations of the phase boundaries 1 and 2; |
| $\bar{\xi}_1$, $\bar{\xi}_2$ | positions of the neutral lines; |
| $\eta$ | $y/l$ dimensionless coordinate; |
| $\mu$ | the Poisson's ratio; |
| $\sigma$, $\sigma_x$, $\sigma_y$ | compressive stress tensor and its components [Pa]; |
| $\tau$ | dimensionless time; |
| $\tau_{xy}$ | components of shear stress tensor [Pa]; |
| $\kappa$ | $h/l$, that is much less than unity; |
| $\chi$ | the thermal diffusivity. |

## References

1. L.Z. Prandtl, *Anwendungsbeispiele zu einem Henckyschen Satz über das plastische Gleichgewicht*, Gesammelte Abhandlungen (Springer-Verlag, Berlin, 1961).
2. L.M. Kachanov, *Basis of the theory of plasticity* (Moscow, 1969).
3. V.D. Blank, *et al.* Sov. Phys. JETP **60**, 525–527 (1984).
4. V.B. Begoulev, *et al.* Sov. Phys. Solid State **31**, 1422-1423 (1989).
5. V.I. Levitas, J. Phys. IV France **5**, C2–41–46 (1995).
6. Y. Boguslavskii, Sov. Phys. Crystall. **37**, 146–149 (1992).
7. A.L. Roitburd, *Problems in contemporary crystallography* (Moscow, 1975).
8. S. Glasstone, *et al.*, *The Theory of Rate Processes* (McGraw-Hill, New York, 1941).
9. A.F. Andreev, Y.V. Sharvin, Soviet Phys. JETP **26**, 865–867 (1967).
10. G.B. Whitman, *Linear and nonlinear waves* (John Wiley and Sons, New York, 1974).