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Iron shielded MRI optimization

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Abstract. The design of the main current systems of an actively shielded and of an iron shielded MRI device for nuclear resonance imaging, is considered. The model for the analysis of the magnetic induction produced by the current system, is based on the combination of a Boundary Element technique and of the integration of two Fredholm integral equations of the first and the second kind. The equivalent current magnetization model is used for the calculation of the magnetization produced by the iron shield. High field uniformity in a spherical region inside the device, and a low stray field in the neighborhood of the device are required. In order to meet the design requirements a multi-objective global minimization problem is solved. The minimization method is based on the combination of the filled function technique and the (1+1) evolution strategy algorithm. The multi-objective problem is treated by means of a penalty method. The actively shielded MRI system results to utilize larger amount of conductor and produce higher magnetic energy than the iron shield device.

Résumé. On veut étudier le projet du système des courants principaux d'un MRI à écran en fer et d'un MRI à écran actif. Le modèle d'analyse du champ magnétique produit par le système de courants est basé sur la combinaison d'une technique Boundary Element et de l'intégration de deux équations intégrales de Fredholm de première et de seconde sorte. On utilise pour calculer la magnétisation produite par l'écran en fer le modèle à courants de magnétisation équivalents. On exige une élévation uniforme du champ dans une région sphérique au cœur de l'appareil et un bas champ magnétique dispersé à proximité de l'appareil. Dans le but de répondre aux impératifs du projet, on va résoudre un problème multiobjectif de minimisation globale. On utilise une technique de minimisation obtenue par la combinaison des méthodes "Filled Function" et "(1 + 1) Evolution Strategy". Le problème multiobjectif est traité avec la méthode des pénalités. Il s'ensuit que le MRI à écran actif utilise une plus grande quantité de conducteur et produit une énergie magnétique plus élevée que le MRI à écran en fer.

PACS. 41.20.-q Electromagnetic fields; electron and ion optics – 41.20.Gz Magnetostatics; magnetic shielding, magnetic induction, boundary-value problems – 87.59.Pw Magnetic resonance imaging (MRI)

1 Introduction

The problem considered in this paper is the determination of the main current system of an iron shielded MRI device for magnetic resonance imaging. The problem consists in the production of a uniform magnetic induction field in a spherical region at the center of the device. In MRI devices the diameter of this region is about 0.5 m and the field strength is between 0.5 and 2 T. Outside of it the field has to be below a legally allowed value.

The determination of the current system for the production of the magnetic field mentioned above, is an inverse problem. Moreover it is a non-stable, non-analytic and ill-posed problem according to the Hadamard definition [1–3]. As the field has to be uniform and the current distribution has to be obtained by a finite number of coils of finite dimensions, the problem has no exact solution. However, an infinity of quasi-solutions, corresponding to non-zero residuals of the equation which relates magnetic field and currents, exists. The choice among them originates a synthesis problem. Different approaches to the solution of the problem have been suggested. All methods consider the solution of the direct problem, which consists in the determination of the field produced by a given current system, evaluate the deviation of the solution from the desired one and, consequently, adjust the current system [4,5]. To do this, several optimization algorithms have been considered. Moreover several criteria for evaluating the deviation from the optimal solution have been adopted. The function to minimize has generally a gorge behavior with several relative minima. Stochastic global minimization methods appear to be the most adequate for the solution of this problem [6–8].

In order to keep the magnetic induction strength outside the device (stray field) below the desired value, active or passive shielding is utilized. Active shielding [9–12] is realized by adding some coils outside the main field coils.

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The electric currents flowing through these coils produce a magnetic induction field which has to balance the main MRI field outside the device. Kalafala in [10] outlines the advantages of this shielding system. In passive shielded devices an iron yoke containing the main field coils is used [4, 5, 13, 14]. In order to calculate the effect of the iron shield Pissanetsky in [5] presents a hybrid method in which a finite element method for the calculation of the magnetization in iron yoke, is utilized. In [13] and [14] the equivalent magnetization current method is used. This method assumes that an equivalent current flow in the iron yoke due to magnetization of iron. This current, as in the active shield, produces a magnetic induction field which balances the main field outside the device. This method appears to be of an easy utilization and estimates the field homogeneity and the stray field with a high accuracy. Moreover the non-linear magnetic properties of iron can be taken into account too. The problem considered by the authors of [13] and [14] is the determination of the current system (main coil dimensions and current intensities) of an iron shielded MRI device producing a highly uniform magnetic induction of a given intensity, inside the device, and a low stray field, outside of it. The iron shield in these works is considered to be fixed.

In the present paper both an actively shielded device and an iron shielded device are considered and their main magnetic systems are calculated. The shielding systems are determined also. In the active case the unknowns of the problem are the coil dimensions and the current intensities of the coils acting as main coils of the MRI device, and the coil dimensions and the current intensities of the coils of the active shielding system. In the iron shielded device the problem unknowns are the coil dimensions and the current intensities of the MRI main coils, and the dimensions of the iron shield. The results obtained for the two systems are compared and discussed. A linear combination of the field non-uniformity in the spherical region of interest and of the stray field outside the device is taken as the objective function of the minimization [11] utilized for the solution of the design synthesis problem. The equivalent magnetization current method is used for the solution of the magnetic field problem in the iron shield case. This method considers an equivalent current flowing in the iron shield due to the magnetization of iron. The equivalent current generates a magnetic induction field which is superimposed to the field generated by the coil currents. The evolution strategy method [15] combined with the filled function search acceleration technique [8] is utilized as minimization algorithm for the solution of the global optimization problem.

In order to reduce the numerical heaviness of the problem and the computation time, the main magnetic induction of the MRI device is considered to be 1 T. For this case linearity of the magnetic properties of iron may be assumed. This assumption is not essential for the utilization of the method described in the present paper. For a magnetic induction larger than 1 T, the iron shield can be considered as a set of solenoidal elements and for each of them the equivalent magnetization current method can be utilized [14]. The number of equations and of unknowns increases with the number of solenoids into which the iron yoke is divided. Hence the numerical heaviness of the problem becomes larger.

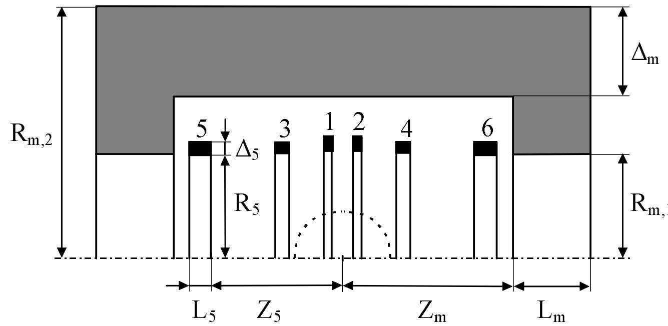

Fig. 1. Iron shielded configurations.

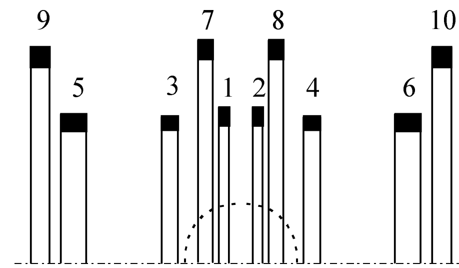

Fig. 2. Actively shielded configuration.

2 MRI magnetic system

The main magnetic system for an MRI device consists of coaxial and symmetrical coils which have to produce a field with a high degree of homogeneity in a spherical region in the center of the device. In the case considered here, the diameter of this region is 0.5 m and the field strength is 1.0 T. In order to evaluate the stray field outside the device, a cylinder with a diameter of 6 m and a height of 8 m, containing the MRI device, is considered. On the surface of the cylinder the field strength has to be below the legally allowed value. The passive shielding scheme is shown in Figure 1. Here an iron shield containing the coils, is used. The active shielding scheme is shown in Figure 2. It is realized with four coils outside the main magnetic system of the MRI. These coils have to produce a field which balances the main field in the outside region where the total magnetic induction field has to be limited. When adopting the equivalent magnetization current method an equivalent current, due to the iron magnetization, is assumed to flow inside the iron yoke. This current produces a magnetic induction outside the yoke, which contributes to the total magnetic induction. The electromagnetic problem formulation for the case of the active shield has already been presented in [8] and [11]. The formulation of the problem for the passive shielding is a modification of that formulation and will be presented in the following.

3 Electromagnetic problem formulation

In this paper the equivalent magnetization current method is adopted for field computation. The magnetization field of the ferromagnetic shield is considered to produce equivalent currents. In the whole space there are three regions of interest: the volume of the ferromagnetic shield τ_m, the coils volume τ_c and the region where the high field homogeneity is required τ_i. The magnetic induction B is given by

$$\nabla \times B = \mu_0 (J + J_m), \tag{1}$$

where $J_m = \nabla \times M$ is the equivalent current density, M is the magnetization and J is the current density in the coils. Both J_m and M are defined in the volume τ_m. J is defined in the volume τ_c. From the solution of equation (1) the relation between the magnetic induction and the current densities results to be:

$$\begin{aligned} B = &\frac{\mu_0}{4\pi}\int_{\tau_c} J(r')\frac{r-r'}{\|r-r'\|^3}dr' \\ &+ \frac{\mu_0}{4\pi}\int_{\tau_m} J_m(r')\frac{r-r'}{\|r-r'\|^3}dr' \\ &+ \frac{\mu_0}{4\pi}\int_{\partial\tau_m} K_m(r')\frac{r-r'}{\|r-r'\|^3}dr', \end{aligned} \tag{2}$$

where $K_m = M \times n$ is the equivalent surface current density [16]. The unit vector n is defined on the surface of the ferromagnetic shield $\partial\tau_m$ and is perpendicular to it.

When linear, isotropic and homogeneous ferromagnetic material is considered, the relation between the magnetization M and magnetic induction B is given by

$$B = \mu_0 \left(1 + \frac{1}{\chi_m}\right) M, \tag{3}$$

where the magnetic susceptibility χ_m is assumed to be constant. As a consequence, from the definition of the equivalent current density it results that $J_m = 0$. On $\partial\tau_m$, from equation (3) and from the definition of K_m, the following relation is obtained:

$$B \times n = \mu_0 \left(1 + \frac{1}{\chi_m}\right) K_m. \tag{4}$$

Hence equation (2) becomes

$$\begin{aligned} &\frac{1}{4\pi}\int_{\tau_c}\left(J(r')\frac{r_m-r'}{\|r_m-r'\|^3}\right) n(r_m)dr' = \\ &-\frac{1}{4\pi}\int_{\partial\tau_m}\left(K_m(r')\frac{r_m-r'}{\|r_m-r'\|^3}\right) n(r_m)dr' \\ &+\left(1+\frac{1}{\chi_m}\right) K_m(r_m), \end{aligned} \tag{5}$$

where r_m belongs to the surface of the iron shield $\partial\tau_m$, border of τ_m. For the unknown function K_m, equation (5) has the form of a Fredholm integral equation of the second kind. It can be written in the following compact form:

$$L_1(J) = N_1(K_m), \tag{6}$$

where $N_1(\cdot)$ and $L_1(\cdot)$ indicate the operators defined by the right-hand member and the left-hand member of equation (5) respectively. Equation (6) states the relation between the current density in the coils and the equivalent surface current density on the surface of the iron shield.

The magnetic induction field within the spherical region τ_i in the center of the MRI device is uniquely determined by its tangential component on the spherical region border $\partial\tau_i$. The following equation is obtained:

$$\begin{aligned} &\frac{\mu_0}{4\pi}\int_{\tau_c} J(r')\frac{r_i-r'}{\|r_i-r'\|^3}t(r_i)dr' \\ &+ \frac{\mu_0}{4\pi}\int_{\partial\tau_m} K_m(r')\frac{r_i-r'}{\|r_i-r'\|^3}t(r_i)dr' = B(r_i)t(r_i), \end{aligned} \tag{7}$$

where $r_i \in \partial\tau_i$ and t is the unit vector tangent to $\partial\tau_i$. Equation (7) has the form of a Fredholm integral equation of the first kind. Equation (7) can be written in the form:

$$L_0(J) + N_0(K_m) = b \tag{8}$$

where $L_0(\cdot)$ and $N_0(\cdot)$ indicate the operators defined by the first term and the second term in the left-hand member of equation (7) respectively. $L_0(J)$ states the contribution of the coil current to the magnetic induction field. $N_0(K_m)$ is the contribution of the iron yoke. b indicates the right-hand member of equation (7). A solution of equation (8) must satisfy equation (6) too. The inverse problem presented by equations (6) and (8) is not stable [17]. Furthermore it is an ill-posed problem according to the Hadamard definition [18]. For the system configuration considered, a solution of it does not exist. A quasi-solution [17] of the problem corresponds to a non-zero residual of the equations (6) and (8) and is a function of the unknowns of the problem. An infinity of quasi-solutions can be found [2].

In the numerical formulation of the equations (6) and (8) Boundary Element technique [19] is utilized. In order to do this, the surface of the iron yoke is split into annular elements where the superficial current density K_m flows. $N_0(K_m)$ and $N_1(K_m)$ are defined on these surface elements. The terms $L_0(J)$ and of $L_1(J)$ are calculated by means of elliptical integrals.

When non-linearity has to be taken into account, the iron yoke is considered to be made of small solenoidal elements. For each element a surface equivalent current is derived and the contribution of it to the magnetic induction field is calculated. In this case for each element a different value of the magnetic susceptibility χ_m is utilized. The value of χ_m used depends on the average magnetic induction in the element.

4 Synthesis problem

As discussed in the previous section, the inverse problem has an infinity of quasi-solutions. The choice of one of these solutions defines a synthesis problem which is solved by means of an optimization technique. In order to do this a cost function P is introduced. The minimization of the cost function leads to the solution required:

$$\begin{aligned} & min \quad P(J) \\ & subject\ to \begin{cases} L_0(J) + N_0(K_m) = b \\ L_1(J) = N_1(K_m). \end{cases} \end{aligned} \tag{9}$$

In order to meet the requirements of the problem, the cost function P has to consider two objectives. The multi-objective optimization problem is solved by defining P as a linear combination of the field non-uniformity and of the stray field outside the device

$$P(J) = c_1 \frac{\Delta B}{B} + c_2 B_s. \tag{10}$$

Here $\Delta B/B$ expresses the magnetic induction non uniformity, B_s is the stray field. Both $\Delta B/B$ and B_s are functions of the unknown J. c_1 and c_2 are penalty constants. The value of $\Delta B/B$ is given by

$$\frac{\Delta B}{B} = \frac{B_{max} - B_{min}}{B_0}, \tag{11}$$

where B_{max} and B_{min} are the maximal and the minimal value of the magnetic induction on the surface $\partial \tau_i$ of the spherical region and B_0 is the magnetic induction value at the center of this region. B_s is the maximal value of the stray field on the surface of the cylinder outside the device as mentioned above.

The minimization procedure adopted in the present work, is based on an evolution strategy method [15] coupled with the filled function search acceleration technique [8]. As a non-analytic, non-stable problem is considered, a stochastic minimization algorithm is utilized. Therefore the evolution strategy algorithm obtained from its $(1+1)$ formulation (in each iteration a parent generates a child) is applied [15]. The filled function technique is used to accelerate the minimization search procedure. When utilizing a deterministic gradient based minimization algorithm, the filled function technique allows one to move down through basins of attraction of decreasing relative minimums toward the global minimum [20]. The evolution strategy algorithm is stochastic and its minimization process does not always lead to a minimum of the basin of attraction to which the minimization starting point belongs. However, as shown in [8] and [11], the filled function technique enhances the efficiency of the global minimization procedure.

5 Comparison of the results

The system configurations considered are those described in Sect. 2. The value of the magnetic induction inside the spherical region τ_i, is 1.0 T and the maximal value allowed for the stray field is 5×10^{-4} T. Furthermore the value of the current density is constrained to be lower than 5×10^8 A/m^2. This constraint is taken into account as superconducting coils based on NbTi technology are considered.

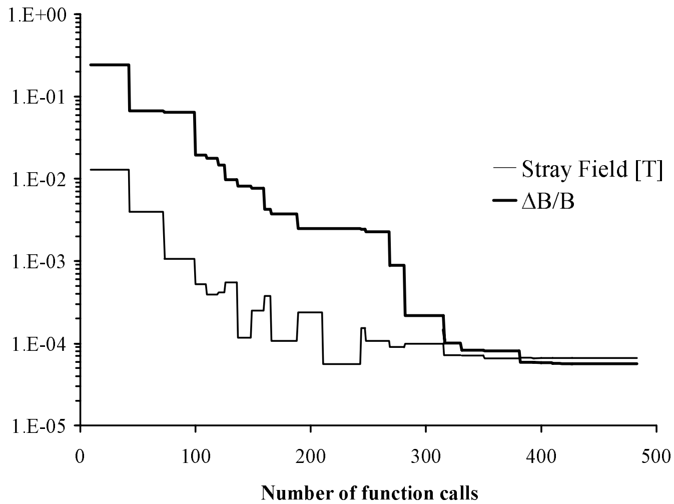

Fig. 3. Behavior of the field non-uniformity $\Delta B/B$ and of the stray field B_s during the last minimization process for the actively shielded configuration.

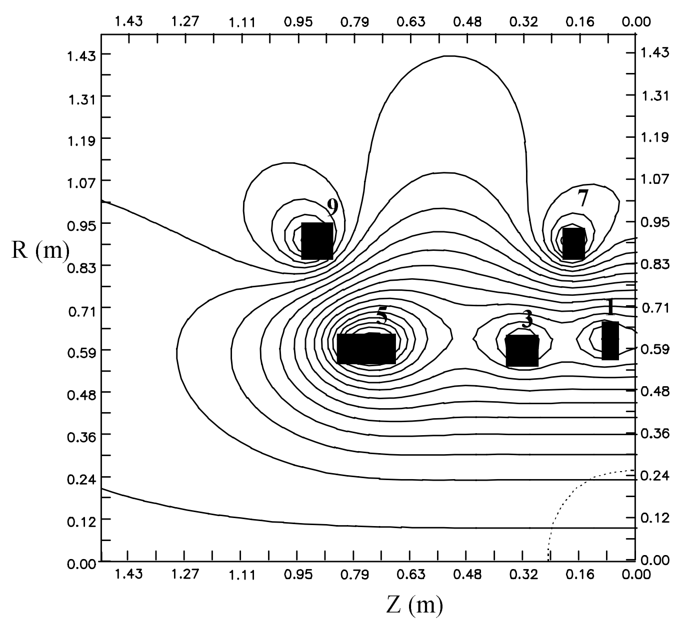

Fig. 4. Coil positions and dimensions and magnetic flux line distribution for the actively shielded configuration.

For the active shield case, the unknowns of the optimization problem are the positions and the dimensions of the coils and the current intensities. To determine the penalty constants c_1 and c_2, some minimization procedures with different values of c_2/c_1 have been performed [11]. The ratio c_2/c_1 for which results in accordance with the requirements are obtained, is 10^{-1}. In order to see whether the minimum obtained is the global one, few minimization procedures utilizing the starting points indicated by the filled function technique have been performed. The behaviors of the field non-homogeneity and of the stray field during the last minimization procedure, before the global minimum is reached, are shown in Figure 3. The numbers of function calls to reach the minimum for the

Table 1. Main results for the actively shielded device.

k	I_k (kA)	Δ_k (m)	L_k (m)	R_k (m)	Z_k (m)
1, 2	313.52	0.0822	0.0469	0.5705	0.0659
3, 4	379.92	0.0787	0.0792	0.5556	0.2711
5, 6	896.10	0.0776	0.1604	0.5558	0.6473
7, 8	−358.28	0.0758	0.0631	0.8451	0.1498
9, 10	−370.15	0.0913	0.0842	0.8447	0.8416

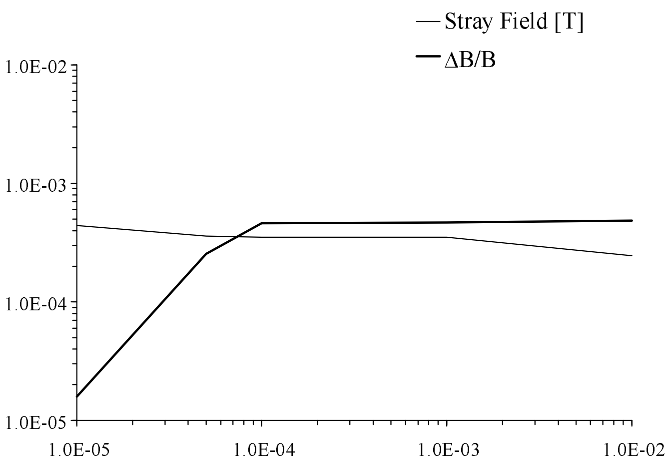

Fig. 5. Values at the minima of the field non-uniformity and of the stray field as functions of the penalty constant ratio c_1/c_2.

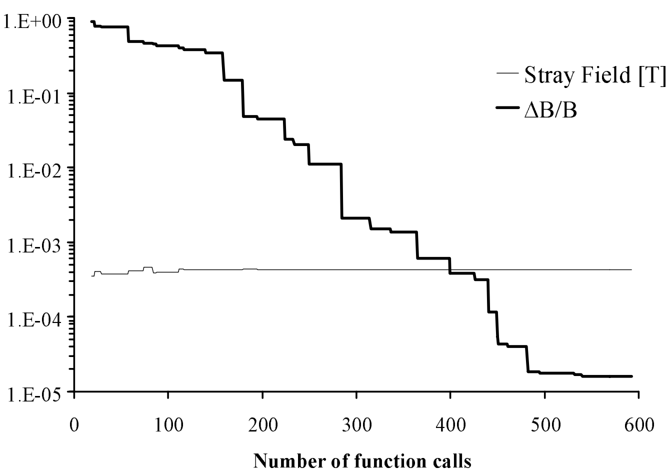

Fig. 6. Behavior of the field non-uniformity $\Delta B/B$ and of the stray field B_s during the last minimization process for the iron shielded configuration.

minimization of B_s and of $\Delta B/B$ appear to be of the same order. The currents intensities, the coil dimensions and their position resulting from the optimization procedure are reported in Table 1. This results correspond to a minimum of the objective function with $B_s = 6.6\times10^{-5}$ T and of $\Delta B/B = 5.7\times10^{-5}$ respectively. The magnetic flux line distribution are shown in Figure 4.

For the iron shield device, the unknowns of the design optimization procedure are the positions and the dimensions of the coils, the current intensities flowing through

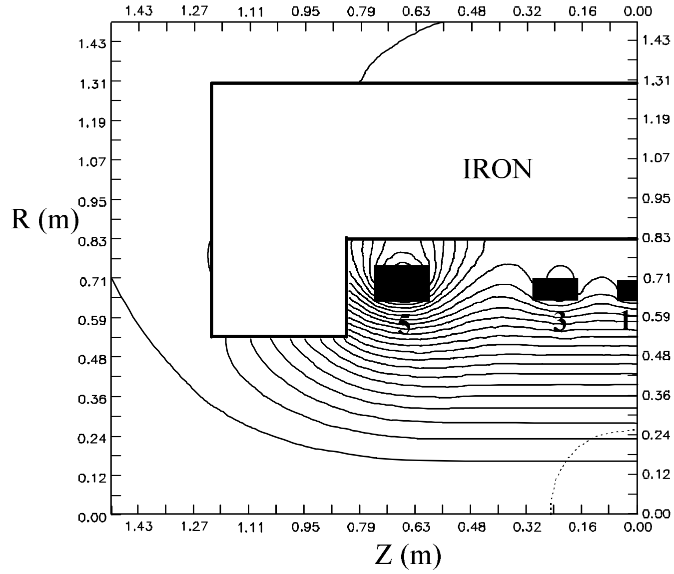

Fig. 7. Coil positions and dimensions and magnetic flux line distribution for the iron shielded configuration.

Table 2. Main results for the iron shielded device.

k	I_k (kA)	Δ_k (m)	L_k (m)	R_k (m)	Z_k (m)
1, 2	59.991	0.0602	0.0610	0.6443	0.0023
3, 4	233.93	0.0721	0.0985	0.6384	0.1557
5, 6	512.70	0.0995	0.1461	0.6247	0.6198
$m, 1$	-	0.473	0.381	0.552	0.815
$m, 2$	-	0.473	0.381	1.305	0.815

them, the ferromagnetic shield dimension and its position. In order to determine the ratio c_2/c_1 for the objective function of the multi-objective minimization some minimization procedures have been performed. In Figure 5 the values of the field non-uniformity and of the stray field as functions of c_2/c_1 are plotted. The behavior of B_s shows a slow decrease. $\Delta B/B$ is increasing for increasing values of c_2/c_1. As the value of the stray field is lower than that legally allowed [11] in the range analyzed, the value of c_2/c_1 considered is that corresponding to the lower minimum of $\Delta B/B$. Therefore c_2/c_1 utilized in the following is 10^{-5}. The behaviors of the field non-homogeneity and of the stray field in the last minimization process are shown in Figure 6. In this case the procedure for minimization of the field non-homogeneity appears to dominate the optimization process. The minimum stray field is reached after few function calls. This minimal values of B_s and of $\Delta B/B$ are 4.2×10^{-4} T and 1.6×10^{-5} respectively. The main results of the design problem are reported in Table 2 and in Figure 7.

In Table 3 the quantities which characterize the two shielding configurations are compared. The Conductor Amount is defined as

$$C.A. = \int_{\tau_c} \|J(r)\| dr$$

Table 3. Comparison of the two configurations.

	Active shield	**Iron shield**
Shield volume (m^3)	-	9.503
C.A. ($MA \cdot m$)	20.0	6.86
Conductor volume (m^3)	0.303	0.215
Magnetic Energy (MJ)	4.474	1.490

and is expressed in MA m. The magnetic energy, the conductor amount and the conductor volume of the iron shielded device are considerably lower than those of the active shield. However a large amount of iron is utilized for the shield. The calculation is performed for a field strength of 1.0 T. For high magnetic fields, as those required in the MRI devices for spectroscopy, the iron shielding is much less effective due to the saturation of the ferromagnetic material. The actively shielded configuration can afford any peak field without losing its shielding effectiveness. Moreover the field strength can assume any intermediate value without degradation in homogeneity.

6 Conclusion

The design problem of the main electromagnetic system of a MRI device for magnetic resonance imaging with an actively shielded configuration and with a passively shielded configuration, has been considered. The requirements of the problem are the need to minimize both the field non-uniformity and the stray field. The problem has been formulated as a multi-objective minimization problem. The results obtained for the two configurations, are presented and compared.

The conductor volume and the magnetic energy of the device is considerably larger in the actively shielded case. However for the case considered with a magnetic induction of 1.0 T in the spherical region of interest, the iron shield requires a large amount of iron. For magnetic resonance spectroscopy, where high magnetic fields are used (typically $B \sim 14$–18 T), the effectiveness of the iron shielding is largely reduced. Moreover the uniformity of the field in the spherical region of interest is degraded when intermediate field strengths are realized. For this case a hybrid approach with actively shielding coils coupled to an iron shielding seems to be the most effective configuration.

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