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Experimental characterization of immersed targets by polar decomposition of the Mueller matrices

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Abstract. The possibility of characterizing immersed targets in laboratory conditions, whatever the water surface roughness is discussed in this paper. The first idea was to reduce the interface effects or better to suppress them within the analysis.We used the Mueller formalism to characterize targets in the laboratory. In optics, each element can be described by a Mueller matrix. Then, in the present work we tried to decompose the global measured matrix and to separate the interface matrix from the target matrix which is the one of interest. This paper shows it impossible to perform such a decomposition. We succeeded in reducing the interface effects without eliminating them entirely. So this paper focuses on finding a decomposition algorithm of a Mueller matrix to deduce the polarimetric type of a target whatever the interface state.

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1 Introduction

This paper is aimed at identifying and classifying immersed targets whatever the air-water interface. The effects of surface roughness on the characterization of different targets were studied in a tank by polarimetry. Small waves, which are the most disturbing for the laser beam, were produced by a wind device simulating a 3 m/s windspeed. Each immersed target was characterized by an experimental 4×4 real Mueller matrix.

The first section of this paper introduces the set-up and the experimental methodology used. The results obtained with two different targets are then presented. Their polarimetric types were obtained by treating their Mueller matrices with a specific algorithm. The fourth section describes an attempt to process data and reduce the disturbances induced in target characterization by the passage through the air-water interface.

2 Experimental methodology

2.1 Experimental set-up

Figure 1 shows the reflexion polarimetric used, composed of the three following parts:

– The first one produces a polarimetric coded signal and consists of:
  - a light source: a continuous argon laser emitting at 514 nm;
  - a polarizing system composed of a vertical linear polarizer P_1 and a quarter-wave plate L_1.

– The second one is the studied target: Targets were immersed in a tank (2 m × 1 m × 0.8 m) and were studied under different experimental configurations. A simulated-wind blowing device could be used to create a disturbed air-water interface.

– The last part, at the output, is used for polarimetric analysis detection:
  - a polarizing analyzer (L_2, P_2),
  - a CCD camera used as detector, connected to a computer by an image processing and acquisition board.

  The two quarter-wave plates, L_1 and L_2, are rotated by step-by-step motors (22.5 degrees in increment and 10^{-2} degree in precision). The two polarizers P_1 and P_2 were initially crossed to obtain a null intensity as the initial position.

2.2 Principle of measurements

One needs to find the Mueller matrix of the studied medium. Let $[\boldsymbol{\varepsilon}]$ be the Stokes vector of the incident light and $[\boldsymbol{\varepsilon}']$ the Stokes vector after crossing the medium, then:

$$[\boldsymbol{\varepsilon}'] = [M]\,[\boldsymbol{\varepsilon}] \quad , \tag{1}$$

$M = [m_{ij}] = 4 \times 4$ matrix of the medium (i, $j = 0...3$). Our set-up requires the use of the Mueller matrices of the

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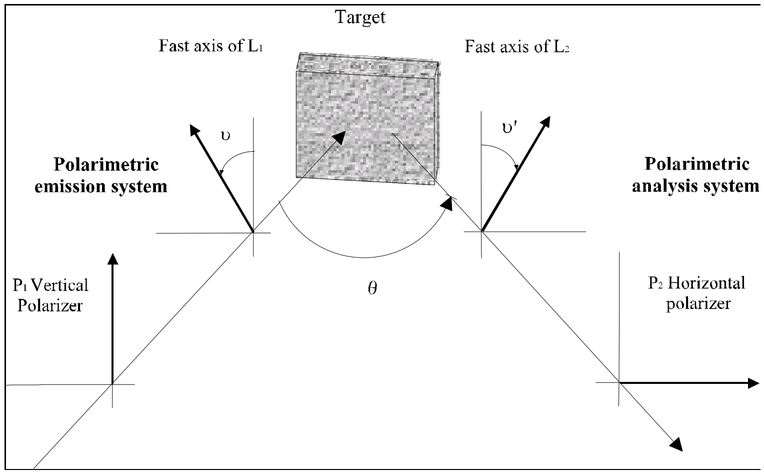


Fig. 1. Reflexion polarimeter.

different polarizing devices $[P_1]$, $[L_1]$, $[L_2]$, $[P_2]$. $[\varepsilon_s]$ the emergent Stokes vector from the polarimetric analyzer is:

$$[\varepsilon_s] = [P_2]\,[L_2]\,[M]\,[L_1]\,[P_1]\,[\varepsilon_e] \quad . \tag{2}$$

The laser source has a vertical polarization; this vertical direction is taken as reference.

$$\varepsilon_e = \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

$$P1 = \frac{1}{2}\begin{bmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \qquad \text{Vertical linear polarizer}$$

$$L_1 = \begin{bmatrix}1&0&0&0\\0&C^2&CS&-S\\0&CS&S^2&C\\0&S&-C&0\end{bmatrix} \qquad \text{with} \begin{cases}C=\cos 2v\\C=\sin 2v\end{cases}$$

$$P_2 = \frac{1}{2}\begin{bmatrix}1&-1&0&0\\-1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \qquad \text{Horizontal linear polarizer}$$

$$L_2 = \begin{bmatrix}1&0&0&0\\0&C'^2&C'S'&-S'\\0&C'S'&S'^2&C'\\0&S'&-C'&0\end{bmatrix} \qquad \text{with} \begin{cases}C'=\cos 2v'\\S'=\sin 2v'\end{cases}$$

v or v' being the orientation of the fast axis of the quarter-wave plate with the vertical reference.

The total transmitted intensity is detected by the CCD camera. The detected signal corresponds to the first element of the Stokes vector $[\varepsilon_s]$; its intensity can be expressed as:

$$\begin{aligned}I(v,v') &= m_{00} + m_{01}C^2 + m_{02}CS + m_{03}S\\ &+ (m_{10} + m_{11}C^2 + m_{12}CS + m_{13}S)(-C'^2)\\ &+ (m_{20} + m_{21}C^2 + m_{22}CS + m_{23}S)(-C'S')\\ &+ (m_{30} + m_{31}C^2 + m_{32}CS + m_{33}S)(S')\,.\end{aligned} \tag{3}$$

To get $[M]$, all the m_{ij} coefficients (i, j = 0...3) must be assessed; so, sixteen measurements for sixteen different couples (v, v') are needed. In practice, experimental noise is reduced by an overdetermined system [1]. Sixty four images are taken, then sixty four intensities are computed:

$$[I_k] = [P][m_l] \text{ with } k = 0...63$$
$$l = 4i + j \quad .$$

The system is solved by:

$$[m_l] = \left([P]^T\,[P]\right)^{-1}[P]^T\,[I_k] \tag{4}$$

$[\ ]^T$ = transposed matrix, $[P]$ contains the information about the plates orientation.

3 Experimental results

Two targets were used for our experiments; one consisted of polythene and one of a dielectric (named target C because of its clear color). These two targets have been chosen because of their highly depolarizing properties: in this case the polarizing effect of the interface is highlighted. Both were immersed in a tank with different sea surface states, *i.e.* with smooth sea, or under cross-wind or upwind conditions (with a 3 m/s windspeed). The detection was made under grazing incidence with a continuous ionized argon laser emitting at 514 nm.

For each experiment, different polarization states for emission and reception were used. So, by using the Stokes-Mueller formalism, the measured intensities gave a 4×4

Table 1. Experimental Mueller matrices for the polythene target.

In the air	**In the smooth sea**
$\begin{bmatrix} 1.000 & -0.003 & 0.005 & -0.003 \\ -0.005 & 0.279 & 0.011 & 0.002 \\ -0.010 & 0.016 & -0.277 & -0.002 \\ 0.005 & 0.000 & 0.000 & -0.276 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 0.277 & -0.032 & -0.006 \\ 0.262 & 0.098 & -0.007 & -0.001 \\ 0.028 & 0.007 & -0.029 & 0.002 \\ -0.015 & -0.005 & -0.003 & -0.025 \end{bmatrix}$
$\sum m_{ij}^2 = 1.231$	$\sum m_{ij}^2 = 1.158$
$P_d = 0.278 \;\; \sigma P_d = 0.003$	$P_d = 0.230 \;\; \sigma P_d = 0.003$
Up-wind	**Cross-wind**
$\begin{bmatrix} 1.000 & 0.170 & -0.064 & 0.020 \\ 0.206 & 0.145 & 0.003 & 0.027 \\ 0.051 & -0.011 & -0.172 & -0.013 \\ -0.006 & 0.002 & -0.019 & -0.138 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 0.247 & -0.058 & -0.004 \\ 0.245 & 0.198 & -0.029 & -0.006 \\ 0.041 & 0.018 & -0.175 & 0.004 \\ -0.020 & 0.005 & 0.003 & -0.138 \end{bmatrix}$
$\sum m_{ij}^2 = 1.149$	$\sum m_{ij}^2 = 1.217$
$P_d = 0.223 \;\; \sigma P_d = 0.016$	$P_d = 0.269 \;\; \sigma P_d = 0.009$

Table 2. Experimental Mueller matrices for target C.

In the air	**In the smooth sea**
$\begin{bmatrix} 1.000 & 0.003 & 0.009 & -0.001 \\ -0.010 & 0.432 & 0.023 & -0.005 \\ -0.011 & 0.028 & -0.427 & 0.002 \\ 0.006 & -0.003 & -0.005 & -0.339 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 0.328 & -0.030 & -0.001 \\ 0.301 & 0.306 & 0.010 & -0.009 \\ 0.047 & -0.000 & -0.199 & -0.000 \\ -0.005 & -0.011 & -0.009 & -0.056 \end{bmatrix}$
$\sum m_{ij}^2 = 1.486$	$\sum m_{ij}^2 = 1.338$
$P_d = 0.402 \;\; \sigma P_d = 0.003$	$P_d = 0.336 \;\; \sigma P_d = 0.005$
Up-wind	**Cross-wind**
$\begin{bmatrix} 1.000 & 0.266 & 0.010 & -0.007 \\ 0.251 & 0.369 & 0.054 & 0.011 \\ 0.012 & 0.074 & -0.325 & -0.015 \\ -0.005 & -0.006 & 0.010 & -0.130 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 0.212 & -0.039 & -0.000 \\ 0.199 & 0.237 & -0.028 & 0.009 \\ 0.019 & 0.011 & -0.204 & -0.002 \\ -0.010 & -0.010 & -0.001 & -0.043 \end{bmatrix}$
$\sum m_{ij}^2 = 1.401$	$\sum m_{ij}^2 = 1.187$
$P_d = 0.366 \;\; \sigma P_d = 0.016$	$P_d = 0.250 \;\; \sigma P_d = 0.012$

real Mueller matrix which characterized both the medium and the target.

Tables 1 and 2 present the experimental Mueller matrices M obtained for polythene and target C. The experimental matrices of the immersed target were compared with the matrix found in the air; this exhibited the evolution of the polarimetric type (Tab. 3).

From the study of the depolarization index, P_d, the studied target is said to be i) not depolarizing, ii) highly or iii) slightly depolarizing. This index P_d was computed for each matrix $[M]$ from the following expression [2]:

$$P_d = \sqrt{\frac{\sum_{ij} m_{ij}^2 - m_{00}^2}{3m_{00}^2}} \qquad 0 \leq Pd \leq 1. \qquad (5)$$

For a pure state of polarization:

$$\text{if } Pd = 0 \Leftrightarrow \sum_{ij} m_{ij}^2 = 1 \quad \text{the medium is totally depolarizing,}$$

$$\text{if } Pd = 1 \Leftrightarrow \sum_{ij} m_{ij}^2 = 4 \quad \text{the medium is not depolarizing.}$$

The variance of this index was given by the following expressions:

$$\sigma^2\left(P_d\right) = \sum_{l=0}^{15} \sigma^2\left(m_l\right) \left(\frac{\partial P_d}{\partial m_l}\right)^2 \qquad (6)$$

Table 3. Polarimetric types.

	In the air	**In the smooth sea**	**Up-wind**	**Cross-wind**
Target C	Partly depolarizing	Partly depolarizing	Partly depolarizing	Partly depolarizing
Polythene	Isotropic depolarizer	Completely depolarizer	Isotropic depolarizer	Isotropic depolarizer

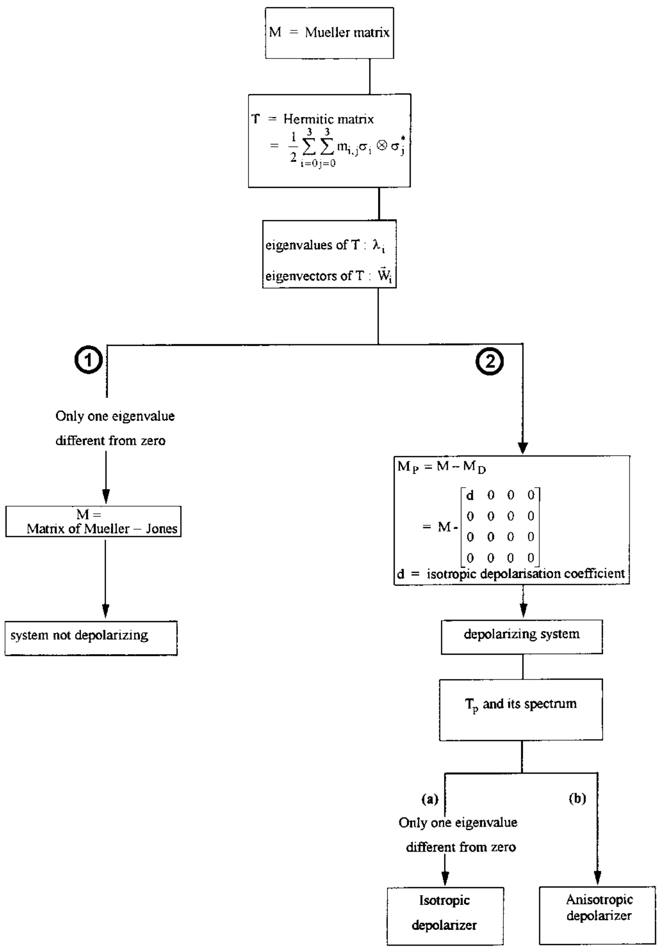


Fig. 2. Algorithm for Mueller matrix treatment.

$$\text{with} \begin{cases} \dfrac{\partial P_d}{\partial m_0} = \dfrac{-l}{m_0} P_d \\ \dfrac{\partial P_d}{\partial m_l} = \dfrac{m_l}{3m_0^2} P_d^{-1} \end{cases} \quad \text{with } l \neq 0.$$

The study of the m_{ij} coefficients of the Mueller matrix gives roughly the isotropic type. So the polythene is said to be isotropic in the air (Tab. 3) because its m_{ii} $(i = 1 \ldots 3)$ coefficients are equal and high enough ($m_{ii} \approx 0.278$) whereas the other terms are negligible.

4 Data processing of the Mueller matrices

4.1 Algorithm of Mueller matrix treatment

The rough study described above was only based on the reading of matrices. Experimental matrices were processed with an algorithm (Fig. 2) specifically elaborated in our laboratory [3] to meticulously classify optical systems according to their depolarizing nature. It enables us to conclude whether a target is depolarizing or not, and especially whether the depolarization is isotropic or anisotropic.

The experimental Mueller matrix M was first used to build an Hermitian matrix T, named coherency matrix. The spectral decomposition of T gives an estimation of M by a Mueller-Jones matrix. Its eigenvalues are then computed; if only one of them differs from zero, *i.e.* there is a prevailing eigenvalue λ_0 and $\lambda_i \approxeq 0$ (for $i = 1, 2, 3$), case ①, then M corresponds to a unique Mueller-Jones matrix characteristic of a non-depolarizing system for completely polarized states. In the other case, case ②, *i.e.* the eigenvalues λ_i, for $i = 1, 2, 3$ are non-zero, the system is depolarizing. Is the depolarization isotropic or not? In fact, it is isotropic when the depolarization of the wave backscattered by the target is the same for any incident pure state. The Mueller matrix can then be decomposed in [4]:

$$M = M_J + M_D \tag{7}$$

where M_J is a Mueller-Jones matrix and M_D is the matrix of an isotropic depolarizer. So, a new matrix, M_P, is computed:

$$\begin{aligned} M_P &= M - M_D \\ &= M - \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{8}$$

where d = isotropic depolarization coefficient.

By carrying out the same analysis on M_P as on M at the beginning of the algorithm, we compute the T_P matrix and its eigenvalues; if only one of them differs from zero (case a) the system is an isotropic depolarizer, if not (case b), it is an anisotropic depolarizer.

The results listed in Table 4 show that the targets keep their polarimetric property: polythene and target C are high depolarizers whatever the experimental configuration. But, their isotropic characteristic differs: polythene is an isotropic depolarizer in the air, but, in smooth sea and under cross-wind conditions, its isotropic property is transformed and the target becomes anisotropic.

This anisotropy results from the different behaviors of the reflection coefficients $r_{//}$ and $r_{\perp}$ at the plane surface of air-water interface similar to a mirror.

So, to find the true polarimetric type of a target whatever its experimental configuration, suppressing the effects of the air-water interface would be worthwhile.

Table 4. Polarimetric type found from study of the experimental Mueller matrix.

	In the air	In smooth sea	Up-wind	Cross-wind
Target C	$\lambda_0 = 1.100$ $\lambda_1 = 0.335$ $\lambda_2 = 0.326$ $\lambda_3 = 0.239$ $\|\Delta M_p\| = 0.075$ $\|\Delta M\| = 0.525$ $\|S\| = 0.017$ ②(b) slightly anisotropic high depolarizer $P_d = 0.402$	$\lambda_0 = 0.994$ $\lambda_1 = 0.424$ $\lambda_2 = 0.309$ $\lambda_3 = 0.273$ $\|\Delta M_p\| = 0.111$ $\|\Delta M\| = 0.591$ $\|S\| = 0.028$ ②(b) slightly anisotropic high depolarizer $P_d = 0.336$	$\lambda_0 = 1.034$ $\lambda_1 = 0.415$ $\lambda_2 = 0.341$ $\lambda_3 = 0.211$ $\|\Delta M_p\| = 0.146$ $\|\Delta M\| = 0.577$ $\|S\| = 0.082$ ②(b) slightly anisotropic high depolarizer $P_d = 0.336$	$\lambda_0 = 0.862$ $\lambda_1 = 0.459$ $\lambda_2 = 0.379$ $\lambda_3 = 0.300$ $\|\Delta M_p\| = 0.113$ $\|\Delta M\| = 0.667$ $\|S\| = 0.064$ ②(b) slightly anisotropic high depolarizer $P_d = 0.250$
Polythene	$\lambda_0 = 0.916$ $\lambda_1 = 0.356$ $\lambda_2 = 0.366$ $\lambda_3 = 0.362$ $\|\Delta M_p\| = 0.006$ $\|\Delta M\| = 0.626$ $\|S\| = 0.018$ ②(a) high isotropic depolarizer $P_d = 0.278$ $D = 0.035$ $\eta = 179.85$	$\lambda_0 = 0.822$ $\lambda_1 = 0.458$ $\lambda_2 = 0.443$ $\lambda_3 = 0.277$ $\|\Delta M_p\| = 0.142$ $\|\Delta M\| = 0.695$ $\|S\| = 0.017$ ②(b) slightly isotropic high depolarizer $P_d = 0.230$	$\lambda_0 = 0.824$ $\lambda_1 = 0.457$ $\lambda_2 = 0.399$ $\lambda_3 = 0.320$ $\|\Delta M_p\| = 0.097$ $\|\Delta M\| = 0.686$ $\|S\| = 0.097$ ②(a) high isotropic depolarizer $P_d = 0.223$ $D = 0.785$ $\eta = 178.72$	$\lambda_0 = 0.898$ $\lambda_1 = 0.414$ $\lambda_2 = 0.383$ $\lambda_3 = 0.305$ $\|\Delta M_p\| = 0.080$ $\|\Delta M\| = 0.641$ $\|S\| = 0.051$ ②(b) slightly anisotropic high depolarizer $P_d = 0.269$

4.2 Influence of the air-water interface

To study the influence of the interface on the matrix, the matrix of a mirror, *i.e.* a non depolarizing optical element, in the air was compared with that of the same mirror immersed in water (Tab. 5). The difference lies in the m_{10} and m_{01} coefficients: $0.4 < m_{10}$, $m_{01} < 0.5$ which differ from zero when the target is immersed. One should notice that the immersed target has the same matrix as a diattenuator. Consequently, the interface creates diattenuation. This effect is noticeable on the Poincare representation in Table 6.

How the Poincare representation should be analyzed?

This representation shows the effects of a Mueller matrix on the state of polarization of light, characterized by its normalized Stokes vector **S** $(S_0 = 1, S1, S2, S3)^T$.

$$S' = MS \quad \text{with } S = \begin{bmatrix} 1 \\ \cos 2\varepsilon \cos 2\vartheta \\ \cos 2\varepsilon \sin 2\vartheta \\ \sin 2\varepsilon \end{bmatrix} \tag{9}$$

S describes the Poincare sphere when $-\pi/4 \le \varepsilon \le \pi/4$ and $0 \le \vartheta \le \pi$. The degree of polarization is defined from:

$$P'(\varepsilon, \vartheta) = \frac{\sqrt{S'^2_1 + S'^2_2 + S'^2_3}}{S'_0} \quad 0 \le P' \le 1 \tag{10}$$

$P' = 1$ when the light is not affected by depolarization, then, the radius of the Poincare sphere is a unit radius. On the contrary, the smaller the radius is, the higher the depolarization.

The isotropic type can also be described with this representation. Depolarization is said to be isotropic when it is identical whatever the input pure state, *i.e.* identical whatever the direction in Cartesian coordinates, the Poincare representation is then a sphere. Conversely, depolarization is said anisotropic when the Poincare figure is an ellipsoid.

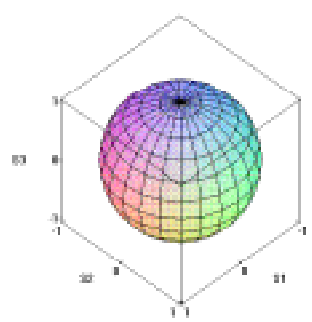
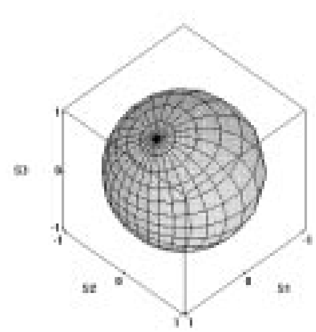
The Poincare representation is then useful to analyse experimental results and describe the polarimetric type

Table 6 shows that the mirror either in the air or immersed in water is a non depolarizing optical element because the

Table 5. Experimental Mueller matrices for a mirror.

In the air	In the smooth sea
$\begin{bmatrix} 1.000 & 0.000 & 0.012 & 0.001 \\ 0.001 & 0.993 & 0.024 & -0.003 \\ -0.009 & 0.017 & -0.989 & -0.007 \\ -0.001 & -0.001 & 0.006 & -0.995 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 0.452 & -0.025 & 0.015 \\ 0.492 & 0.959 & 0.049 & 0.007 \\ 0.047 & 0.072 & -0.867 & -0.022 \\ -0.019 & 0.009 & 0.022 & -0.898 \end{bmatrix}$
$\sum m_{ij}^2 = 3.955$	$\sum m_{ij}^2 = 3.937$
$P_d = 0.992 \;\; \sigma P_d = 0.003$	$P_d = 0.989 \;\; \sigma P_d = 0.007$
Diattenuator	
$\frac{1}{2}\begin{bmatrix} a & \pm b & 0 & 0 \\ \pm b & a & 0 & 0 \\ 0 & 0 & -e & 0 \\ 0 & 0 & 0 & -e \end{bmatrix}$ with $\begin{cases} a & = P_x^2 + P_y^2 \\ b & = P_x^2 - P_y^2 \\ e & = 2P_xP_y \end{cases}$	P_x, P_y amplitude transmission coefficients

Table 6. Sphere of Poincare obtained with a theoretical mirror.

Sphere of Poincare obtained with a theoretical mirror	
In the air	In smooth water
	

Poincare representation is a sphere with a unit radius. As previously indicated the diattenuation is noticeable on this representation: it is characterized by the poles which are getting closer.

Poles represent the circular polarizations. Such a polarization can be decomposed in two linear polarizations, *i.e.* vertical + horizontal. Poles tend toward the vertical polarization which is then better transmitted by the interface.

So, the Poincare representation evidences the different behaviors of the reflection coefficients; the diattenuation created by the interface is displayed by the displacement of the poles.

4.3 Polar decomposition of a Mueller matrix

This section describes an attempt to reduce to a minimum or suppress the effects of the interface. In this aim we propose a polar decomposition elaborated by Chipman and Lu [5]. With their algorithm a Mueller matrix is decomposed into a product of three matrices:

$$M = M_\Delta \, M_R \, M_D \tag{11}$$

M_D = matrix of a diattenuator, M_R = matrix of a retarder, M_Δ = matrix of a depolarizer.

$$M_D = T_u \begin{bmatrix} 1 & \mathbf{D}^T \\ \mathbf{D} & m_D \end{bmatrix} \tag{12}$$

with T_u = transmittance for an unpolarized light, $\mathbf{D}$ = diattenuation vector.

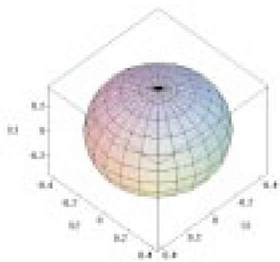
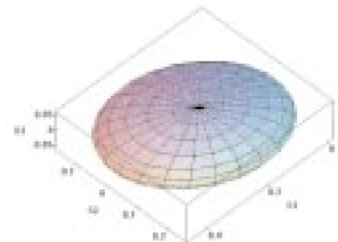
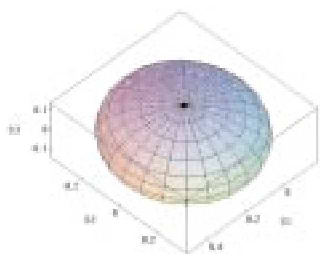
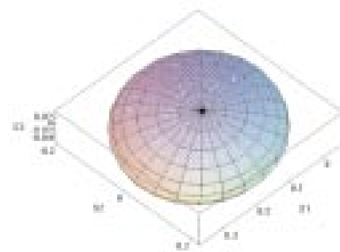
$$M_R = \begin{bmatrix} 1 & \mathbf{O}^T \\ \mathbf{O} & m_R \end{bmatrix} \tag{13}$$

$$M_\Delta = \begin{bmatrix} 1 & \mathbf{O}^T \\ \mathbf{P}_\Delta & m_\Delta \end{bmatrix} \tag{14}$$

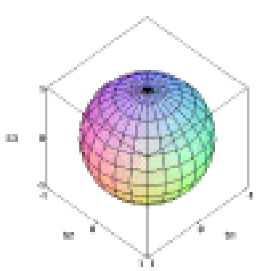
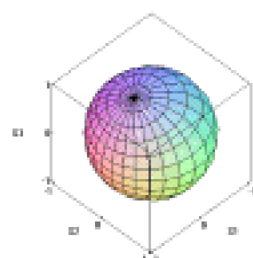
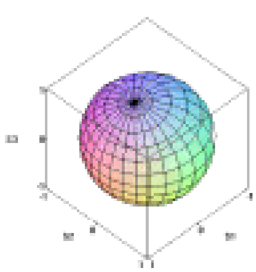
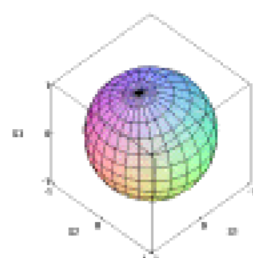
with $\mathbf{P}_\Delta$ = polarizance vector, $\mathbf{O}$ = zero vector.

The interest of this decomposition stands in using a diattenuator matrix. The reflectance is not the same at

Table 7. (a) Depolarization for target C.(b) Diattenuation for target C.

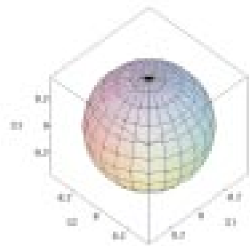
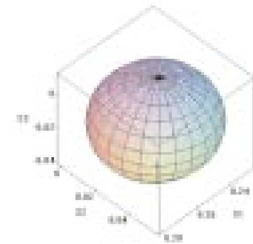
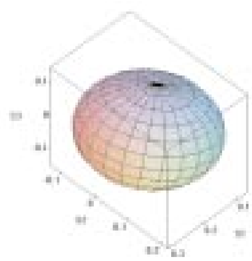
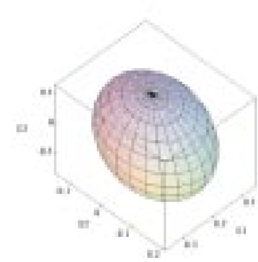
Sphere of Poincare for depolarization	
In the air	**In smooth water**
 $P_{\Delta}^{T} = (-0.01 \quad -0.01 \quad 0)$	 $P_{\Delta}^{T} = (0.23 \quad 0.05 \quad 0)$
Up-wind	**Cross-wind**
 $P_{\Delta}^{T} = (0.16 \quad 0 \quad 0)$	 $P_{\Delta}^{T} = (0.16 \quad 0.01 \quad -0.01)$

(a)

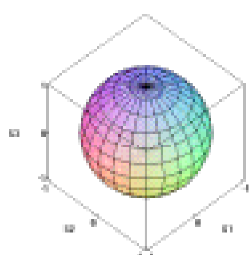
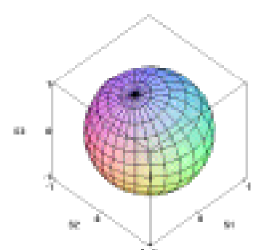
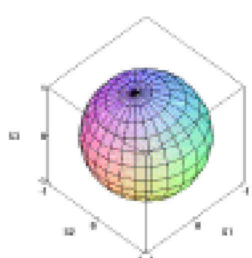
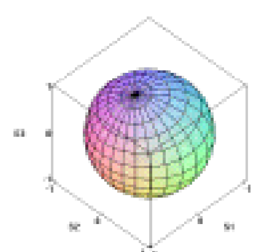
Sphere of Poincare for diattenuation	
In the air	**In smooth water**
 D = 0.01	 D = 0.329
Up-wind	**Cross-wind**
 D = 0.266	 D = 0.215

(b)

Table 8. (a) Depolarization for polythene. (b) Diattenuation for polythene.

Sphere of Poincare for depolarization	
In the air	**In smooth water**
 $P_{\Delta}^{T} = (0 \quad 0 \quad 0)$	 $P_{\Delta}^{T} = (0.25 \quad 0.03 \quad -0.02)$
Up-wind	**Cross-wind**
 $P_{\Delta}^{T} = (0.19 \quad 0.04 \quad -0.01)$	 $P_{\Delta}^{T} = (0.21 \quad 0.03 \quad -0.02)$

(a)

Sphere of Poincare for diattenuation	
In the air	**In smooth water**
 D = 0.006	 D = 0.279
Up-wind	**Cross-wind**
 D = 0.182	 D = 0.254

(b)

Table 9. Polarimetric type found from the M'_Δ study.

	In the air	**In smooth sea**	**Up-wind**	**Cross-wind**
Target C	$\|\Delta M_p\| = 0.075$ $\|\Delta M\| = 0.525$ $\|S\| = 0.017$ ②(b) slightly anisotropic high depolarizer $P_d = 0.402$	$\|\Delta M_p\| = 0.136$ $\|\Delta M\| = 0.734$ $\|S\| = 0.028$ ②(b) slightly anisotropic high depolarizer $P_d = 0.184$	$\|\Delta M_p\| = 0.167$ $\|\Delta M\| = 0.653$ $\|S\| = 0.082$ ②(b) slightly anisotropic high depolarizer $P_d = 0.287$	$\|\Delta M_p\| = 0.135$ $\|\Delta M\| = 0.746$ $\|S\| = 0.064$ ②(b) slightly anisotropic high depolarizer $P_d = 0.171$
Polythene	$\|\Delta M_p\| = 0.006$ $\|\Delta M\| = 0.626$ $\|S\| = 0.018$ ②(a) high isotropic depolarizer $P_d = 0.278$	$\|\Delta M_p\| = 0.003$ $\|\Delta M\| = 0.842$ $\|S\| = 0.017$ ②(a) high isotropic depolarizer $P_d = 0.028$	$\|\Delta M_p\| = 0.097$ $\|\Delta M\| = 0.745$ $\|S\| = 0.097$ ②(a) high isotropic depolarizer $P_d = 0.146$	$\|\Delta M_p\| = 0.036$ $\|\Delta M\| = 0.731$ $\|S\| = 0.051$ ②(a) high isotropic depolarizer $P_d = 0.158$

the air-water interface in TM mode as in TE mode, this interface can then be described by a diattenuator matrix M_D. If this matrix is only related to the interface, the target effects will be separated from the interface effects.

M_R is always the same matrix identical to a mirror, which is quite normal because experiments were made in backscattering. This explains why only M_D and M_Δ were studied.

The diattenuator matrices of the two targets (target C and polythene) were first analyzed in the different configurations studied (Tabs 7b and 8b). If these matrices only characterize the interface, the results obtained must be identical and will not depend on the target nature. Let us consider now a target immersed in smooth sea. D, the diattenuation coefficient, is not the same for target C ($D = 0.329$) as for polythene ($D = 0.279$). Thus, the matrix M_D is not only related to the interface, but it also includes the target effects in the interface diattenuation.

The matrix, M_Δ, must be studied to find the target characteristics. This was carried out by using the Poincare representation given in Tables 7a and 8a with the transposed vector of polarizance, $\mathbf{P}_\Delta$, for the two targets in various configurations. This vector, $\mathbf{P}_\Delta$, only characterizes $\mathbf{OO'} = \mathbf{P}_\Delta$, the displacement of the center of the sphere O $(0, 0, 0)$ and does not deform the Poincare representation. The analysis of the Poincare figures shows that polythene is a high isotropic depolarizer. It is highly depolarizing because the spheres diameters are small ($R \approxeq 0.2$); and isotropic because the Poincare figures are not ellipsoidal but spherical, *i.e.* the depolarization is the same whatever the direction and the incident polarization state. On the other hand, the Poincare representation of target C is no longer a sphere but an ellipsoid, so its depolarization is anisotropic. Consequently, target C is a high anisotropic depolarizer.

As $\mathbf{P}_\Delta$ does not modify the Poincare form, we can make a new matrix, M'_Δ, without this vector:

$$M'_\Delta = \begin{bmatrix} 1 & \mathbf{O}^T \\ \mathbf{O} & m_\Delta \end{bmatrix}. \tag{15}$$

The Mueller matrix M'_Δ is then processed with the algorithm described above to accurately characterize the polarimetric type. The results obtained are given in Table 9. By comparison with Table 4, the isotropic type of the polythene is conserved here whatever the experimental conditions.

This polar decomposition associated with the treatment algorithm of Mueller matrices does not allow thus isolation of the interface effects. However, this treatment always enables the identification of the polarimetric type of a given target whatever the experimental conditions, despite the passage through the air-water interface.

5 Conclusion

The aim of this paper was to characterize in the laboratory immersed targets despite interface effects. In our experiments, Mueller matrices were used and analyzed to discriminate objects, but Section 2 highlighted that the passage through the air-water interface has a great influence on the results. The solution proposed in Section 3 is to decompose the global matrix into a product of matrices: one for the interface, another one for the studied target. In fact, the decomposition showed that is not so simple: it only enabled characterization of the diattenuator on the one hand, and depolarization on the other hand. We did not succeed in characterizing the state of the water surface, but by using this polar decomposition the influence of the interface was reduced. We also demonstrated that knowing the true polarimetric type of the target, despite the air-water interface passage and whatever the experimental conditions, was possible by applying a decomposition algorithm.

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